has

(i) no solution

(ii) unique solution. **2½**

(b) Solve the following system of equation :

$$4x + 5y + 6z = 0$$
$$5x + 6y + 7z = 0$$
$$7x + 8y + 9z = 0$$ **2½**

5. (a) Show that if A is Hermitian and P is unitary, then $P^{-1}AP$ is Hermitian. **2½**

(b) If $A = \begin{bmatrix} 2 & 1 & 2 \\ -3 & -3 & -1 \\ 4 & 1 & 1 \end{bmatrix}$:

(i) Write the quadratic form corresponding to the matrix.

(ii) Rewrite the matrix A of the form so that it is symmetric. **2½**

Section III

6. (a) Solve the equation $4x^4 + 8x^3 + 13x^2 + 2x + 3 = 0$ it being given that the sum of two of its roots is zero. **2½**

Subject Code—52003

B. A. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(First Semester)

MATHEMATICS

BAMH-111

Algebra (B.A. Mathematics)-1st Semester

Time : 3 Hours *Maximum Marks : 25*

Note : Attempt any *Five* questions. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Qustion)

1. (a) Prove that diagonal element of a Hermitian matrix are all real. **1**

(b) If α is eigen value of a non-singular matrix A, then prove that $\dfrac{|A|}{\alpha}$ is an eigen value of adj A. **1**

(c) Show that the equation $x^{2n} - 1 = 0$ has only two real roots. **1**

(d) Find the rank of a matrix : **1**

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

(e) Express the following bilinear forms in matrix notation : **1**

$$2x_1y_1 + 3x_1y_1 + 8x_2y_1 + 7x_2y_2 - 5x_3y_1 - 3x_3y_2$$

Section I

2. (a) Every square matrix can be expressed in one and only one way as a sum of two matrices in which one is hermitian and other is skew-hermitian. **2½**

(b) Find the non-singular matrices P and Q such that PAQ is in normal form, where :

$$A = \begin{bmatrix} 2 & 2 & -6 \\ -1 & 2 & 1 \end{bmatrix}$$ **2½**

3. (a) Verify the Cayley-Hamilton Theorem for matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ and find A^{-1}.

3

(b) Find k if the vectors $\begin{bmatrix} 2 \\ 0 \\ k \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 1 \end{bmatrix}$ are linearly dependent. **2**

Section II

4. (a) For what value of λ, does the system :

$$\begin{bmatrix} -1 & 2 & 1 \\ 3 & -1 & 2 \\ 0 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(b) Find the condition that the roots of equation $x^3 + 3px^2 + 3qx + r = 0$ are in G.P. **2½**

7. (a) If α, β, γ are the roots of $x^3 - x - 1 = 0$, show by transforming technique that : **2½**

$$\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}=-7$$

(b) Find the equation of squared differences of cubic $x^3 + 6x^2 + 7x + 2 = 0$. **2½**

Section IV

8. (a) Solve the equation $28x^3 - 90x^2 + 1 = 0$ by Cardan's method. **2½**

(b) Apply Descarte's method to solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$. **2½**

9. (a) Show that the equation $x^3 + x^2 - 2x - 1 = 0$ has three real roots. **2½**

(b) Solve the equation by Ferrari's method : **2½**

$$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$$

(b) Find the condition that the roots of equation $x^3 + 3px^2 + 3qx + r = 0$ are in G.P. **2½**

7. (a) If α, β, γ are the roots of $x^3 - x - 1 = 0$, show by transforming technique that : **2½**

$$\frac{1+\alpha}{1-\alpha}+\frac{1+\beta}{1-\beta}+\frac{1+\gamma}{1-\gamma}=-7$$

(b) Find the equation of squared differences of cubic $x^3 + 6x^2 + 7x + 2 = 0$. **2½**

Section IV

8. (a) Solve the equation $28x^3 - 90x^2 + 1 = 0$ by Cardan's method. **2½**

(b) Apply Descarte's method to solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$. **2½**

9. (a) Show that the equation $x^3 + x^2 - 2x - 1 = 0$ has three real roots. **2½**

(b) Solve the equation by Ferrari's method : **2½**

$$x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$$