Section III

6. (a) Two of the roots of the equation $9x^4 - 45x^3 + 53x^2 + 5x + 6 = 0$ are equal in magnitude but opposite in sign. Compute the solution of the equation. **2½**

(b) Find the condition that two roots of the equation $x^3 - bx^2 + cx - d = 0$ be equal. **2½**

7. (a) If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are in harmonic progression, show that the mean root is $\frac{3c}{b}$. **2½**

(b) If the roots of the equation $x^3 - 6x^2 + 11x - 6 = 0$ be α, β, γ, find the equation whose roots are $\beta^2 + \gamma^2$, $\gamma^2 + \alpha^2$, $\alpha^2 + \beta^2$. **2½**

Subject Code—52000

B. A. EXAMINATION

(Batch 2017) Re-appear)

(First Semester)

MATHEMATICS

BM-111

Algebra

Time : 3 Hours *Maximum Marks :* 27

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

1. (a) Prove that the diagonal elements of a skew-Hermitian matrix are either purely imaginary or zero. **1½**

(b) Prove that zero is a eigen value of a matrix if and only if A is singular. **1½**

(c) Define Bilinear form with example. **1**

(d) Find the equation whose roots are the squares of the roots of $x^3 + qx + r = 0$. **1½**

(e) Discuss the nature of the roots of $x^3 - 9x - 6 = 0$. **1½**

Section I

2. (a) The necessary and sufficient condition for a square matrix A to be invertible is that A is non-singular. **3**

(b) Express $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ as a product of elementary matrices. **2**

3. (a) State and prove Cayley-Hemilton theorem. **2½**

(b) If $A = \begin{bmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & -1 \end{bmatrix}$, compute matrix B = $3A - 2A^2 - A^3 - 5A^4 + A^5$. **2½**

Section II

4. (a) For what value of λ, does the system $\begin{bmatrix} -1 & 2 & 1 \\ 3 & -1 & 2 \\ 0 & 1 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has :

(i) No solution

(ii) Unique solution. **2½**

(b) Prove that the eigen value of a unitary matrix are of unit modulus. **2½**

5. (a) Show that the matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is orthogonal if and only if $A = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$ or $\begin{bmatrix} a & b \\ b & -a \end{bmatrix}$, where $a^2 + b^2 = 1$. **2**

(b) Show that every Bilinear-form can be reduced to Canonical form. **3**

Section IV

8. (a) Solve the equation $a_0x^4 + 4a_1x^3 + 6a_2x^2 - 4a_3x + a_4 = 0$ by Descarte's method. **3**

(b) Solve the equation $x^3 - 12x - 65 = 0$ by Cardan's method. **2**

9. (a) Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ by Ferrari's method. **3**

(b) If an equation involves no add power of the variable and all its terms are positive, prove that all the roots are complex. **2**

Section IV

8. (a) Solve the equation $a_0x^4 + 4a_1x^3 + 6a_2x^2 - 4a_3x + a_4 = 0$ by Descarte's method. **3**

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9. (a) Solve the equation $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$ by Ferrari's method. **3**

(b) If an equation involves no add power of the variable and all its terms are positive, prove that all the roots are complex. **2**