No. 2231122500 83 Exam Code : M-23

Subject Code—59004

B. A. EXAMINATION

(Main) (Batch 2022 Onwards)

(Second Semester)

MATHEMATICS

BAMH-105

Vector Calculus and Geometry

(B.A. Mathematics)

Time : 3 Hours *Maximum Marks : 25*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory.

1. (a) Show that the vector $\vec{a}, \vec{b}, \vec{c}$ are coplanar if and only if $\vec{b}+\vec{c}, \vec{c}+\vec{a}, \vec{a}+\vec{b}$ are coplanar.

P.T.O.

(b) If S is any closed surface enclosing a volume V and $\vec{f} = x\hat{i} + 2y\hat{j} + 7z\hat{k}$, show that :

$$\iint_S \vec{f}.\hat{n}dS = 10V.$$

(c) To prove that one and only one conic of a confocal system will touch a given straight line.

(d) Find the equation of the tangent to the conic :

$$x^2 + 2xy + y^2 - 2x - 1 = 0 \text{ at } (0, 1)$$

(e) A point moves such that the ratio of its distances from two fixed points is constant. Show that its locus is a sphere.

5×1=5

Unit I

2. (a) If $\vec{a} = \sin\theta\hat{i} + \cos\theta\hat{j} + \theta\hat{k}$, $\vec{b} = \cos\theta\hat{i} - \sin\theta\hat{j} - 3\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} - 3k$, find :

$$\frac{d}{d\theta}\left\{\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right\} \text{ at } \theta = 0$$

8/4

(b) Show that $\operatorname{div}\left[\frac{f(r)\vec{r}}{t}\right] = \frac{1}{r^2}\frac{d}{dr}\left(r^2 f(r)\right)$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. 5

3. (a) Find the angle of intersection at $(4, -3, 2)$ of spheres $x^2 + y^2 + z^2 = 29$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z - 4 = 0$.

(b) Find curl grade r^n when $r = |\vec{r}| = \left|x\hat{i} + y\hat{j} + z\hat{k}\right|$. 5

Unit II

4. (a) Evaluate $\int_C \vec{f}.d\vec{r}$, where

$$\vec{f} = c[-3a\sin^2 t\cos t\hat{i} + a(2\sin t - 3\sin^3 t)\hat{j} + b\sin 2t\hat{k}]$$

and C is given by $\vec{r} = a\cos t\hat{i} + a\sin t\hat{j} + bt\hat{k}$ from $t = \frac{\pi}{4}$ to $t = \frac{\pi}{2}$.

(b) Verify Stoke's theorem for $\bar{f} = x^2\hat{i} + xy\hat{j}$, integrated round the square in the plane $z = 0$ bounded by the lines $x = 0$, $y = 0$, $x = a$, $y = a$. 5

5. (a) If $\bar{f} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ evaluate $\iint_S \bar{f} \cdot \hat{n} \, dS$ where S is the surface of the cube bounded by $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

(b) Verify Green's theorem in plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the closed curve bounded by $x = 0$, $y = 0$ and $x + y = 1$. 5

Unit III

6. Trace the conic $x^2 + 2xy + y^2 - 2x - 1 = 0$. 5

7. (a) Prove that equation of the circle having double contact with the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, at the ends of latus rectum is $x^2 + y^2 - 2ae^3x = a^2(1 - e^2 - e^4)$.

(b) Pair of tangents are drawn to the conic, $\alpha x^2 + \beta y^2 = 1$ so as to be always parallel to the conjugate diameters of the conic $ax^2 + 2hxy + by^2 = 1$. Show that locus of their points of intersection is the conic, $ax^2 + 2hxy + by^2 = \frac{a}{\alpha} + \frac{b}{\beta}$. 5

Unit IV

8. (a) A sphere of constant radius r passes through the origin and meets the axes in A, B, C. Show that locus of the foot of the perpendicular from O to the plane ABC is given by $(x^2 + y^2 + z^2)^2 . (x^{-2} + y^{-2} + z^{-2}) = 4r^2$.

(b) Prove that :

$$4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4 = 0$$

represents a cone whose vertex is $(-1, -2, -3)$. 5

9. (a) Find the equations of the two sphere of the co-axial system $x^2 + y^2 + z^2 - 5 + k(2x + y + 3z - 3) = 0$ which touch the plane $3x + 4y = 15$.

(b) Show that the equation of the quadric cylinder, whose generators intersect the curve $ax^2 + by^2 = 2z$, $lx + my + nz = p$ and are parallel to z-axis is $n(ax^2 + by^2) + 2lx + 2my - 2p = 0$. 5