(b) State and prove fundamental theorem on homomorphism of groups. **2½**

Section III

6. (a) Prove that every finite ring has non-zero characteristic. **2½**

(b) Prove that a field has no proper ideals. **2½**

7. (a) Prove that an ideal I of a commutative ring R with unity is maximal iff R|I is a field. **2½**

(b) Prove that ring of integers is a principal ideal ring. **2½**

Section IV

8. (a) Show that units of $z\ [i]$ are ± 1, $\pm i$. **2½**

(b) Let R be a principal ideal domain which is not a field. Then an ideal $I = \langle a \rangle$ is maximal iff 'a' is an irreducible element. **2½**

Roll No. .... **Exam Code : M-21**

Subject Code—52012

B. A. EXAMINATION

(Batch 2017) (Reappear)

(Fifth Semester)

MATHEMATICS

BM-352

Groups and Rings

Time : 3 Hours *Maximum Marks :* 26

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Give an example of a set which is closed w.r.t. multiplication but not a group. Justify it. **1**

(b) Let $G = \{1, \omega, \omega^2\}$ be the cube roots of unity. Find the order of each element of G w.r.t. binary operation multiplication. **1**

(c) Define Inner automorphism of a group. Give an example of inner automorphism for abelian group. **1**

(d) Give an example of an integral domain which is not a field. **1**

(e) Define characteristic of a ring. What is the characteristic of an infinite field ? **1**

(f) Give an example of a ring having a prime ideal which is not maximal ideal. **1**

Section I

2. (a) If $a \in G$ is an element of order n, then prove that :

$$o\left(a^k\right) = \frac{o(a)}{g.c.d.(n_1 k)} = \frac{n}{g.c.d.(n_1 k)},$$ where

'o' stands for order. **2½**

(b) A subgroup H of G is normal in G iff product of two left cosets of H in G is again left coset. **2½**

3. (a) Prove that if H is the only subgroup of finite order in a group G, then H is normal subgroup of G. **2½**

(b) Give an example to show that union of two subgroups may or may not be a subgroup. **2½**

Section II

4. (a) Prove that every subgroup of a finite cyclic group is characteristic subgroup. **2½**

(b) Find the quotient group S_4/A_4, where S_4 is permutation group of degree 4 and A_4 is alternating group. **2½**

5. (a) Let G be a finite group. If f is an automosphism of G such that $f(x) = x$ for $x \in G$, iff $x = e$, then prove that any $g \in G$ can be expand as $x \in G$ for some $x \in G$. **2½**

9. (a) Show that $f(x, y) = x^3 + y^3 + xy$ is irreducible polynomial over k, the field of complex numbers. **2½**

(b) Prove that $z[\sqrt{-5}]$ is an integral domain but not a UFD. **2½**

9. (a) Show that $f(x, y) = x^3 + y^3 + xy$ is irreducible polynomial over k, the field of complex numbers. **2½**

(b) Prove that $z[\sqrt{-5}]$ is an integral domain but not a UFD. **2½**