Section III

6. (a) Let T : $R^3 \to R^3$ be a linear operator defined by :

$$T(x, y, z) = (2x, 4x - y, 2x + 3y - z).$$

Show that T is invertible and fine T^{-1}. **2½**

(b) If C(R) is the vector space and T is a linear transformation on C(R) define by $T(a + ib) = a - ib$ for all $a, b \in R$; find the matrix of T with respect to the ordered basis B = $\{1 + i, 1 + 2i\}$. **2½**

7. (a) Let B = $\{u_1, u_2, ..., u_n\}$ be ordered basis for U. If T is a linear transformation from U to U then for any $u \in U$, [T(u), B] = [T, B] [u, B.]. **2½**

(b) Prove that if T : V $\to$ V is a linear transformation. Then $\lambda \in F$ is an eigen value of T iff T – $1l$ is singular. Also, the eigen space of 1 is the kernel of T – λl. **2½**

Roll No. .... **Exam Code : J-21**

Subject Code—53902

B. A. EXAMINATION

(Batch 2017)

(Re-appear)

(Sixth Semester)

MATHEMATICS

BM-362

Linear Algebra

Time : 3 Hours *Maximum Marks :* 26

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Let R be the field of real numbers and

$$W = \{(x, y, z) : x + y - z$$
$$= 0.2x + 3y - z = 0, \; x, y, z \in R\}$$

Show that W is subspace of V_3(R). **1.2**

(b) Show that the set $S = \{1, i\}$ which is a subset of vector space C over the field of real numbers generates C over R. **1.2**

(c) Let $T : U(F) \to V(F)$ be a linear transformation. If $u_1, u_2, \ldots\ldots\ldots, u_n$ are linearly independent vectors of U and T is one to one, then $T(u_1), T(u_2), \ldots\ldots\ldots\ldots, T(u_n)$ are also linearly independent. **1.2**

(d) Show that the linear transformation $T : R^4 \to R^3$ such that $T(x, y, z, t) = (2x, y, z+t, 0)$ is a singular transformation. **1.2**

(e) If x and y are vectors in an inner product space V(F), then prove that $x = y$ iff $< x, z > = < y, z >$ for all $z \in V$. **1.2**

Section I

2. (a) Show that the linear sum of two subspaces W_1 and W_2 of a vector space V(F) is a subspace generated by the union of W_1 and W_2, i.e., $W_1 + W_2 = <W_1 \cup W_1>$. **2½**

(b) State and prove Existance Theorem. **2½**

3. (a) If W is a subspace of a finite dimensional vector sapce V(F), then show that $\dim V/W = \dim V - \dim W$. **2½**

(b) If S is linearly independent and $v \notin < S >$, then prove that the set $S \cup \{v\}$ is linearly independent. **2½**

Section II

4. (a) Let $\{u_1 = (1, 1, -1), u_2 = (4, 1, 1), u_3 = (1, -1, 2)\}$ be a basis of R^3. Let $T : R^3 \to R^2$ be the linear transformation such that $T(u_1) = (1, 0)$; $T(u_2) = (0, 1)$, $T(u_3) = (1, 1)$. Find T. **2½**

(b) State and prove Fundamental Theorem of Vector Space Homomorphism. **2½**

5. (a) Find the linear transformation $T : R^3 \to R^3$ whose range space is spanned by the vectors (1, 2, 3), (4, 5, 6). **2½**

(b) If V is a finite dimensional vector space and W be a subspace of V, then prove that A (A(W)) = W. **2½**

Section IV

8. (a) State and prove Triangle Inequality for Inner Product Space. **2½**

(b) Let M and N be subspaces of finite dimensional inner product space V. Then show that : **2½**

$$(M + N)^{\perp} = M^{\perp} \cap N^{\perp}.$$

9. (a) Prove that every finite dimensional inner product space has an orthonormal basis. **2½**

(b) Let T be a linear operator on a unitary space V. Prove that T is normal iff $\|T^*(u)\| = \|T(u)\|$ for all $u \in V$. **2½**