Unit II

4. (a) Discuss the continuity at $x = 0$, for the function $f(x) = \dfrac{x - |x|}{x}, x \neq 0$ and $f(0) = 0$.

(b) For what values of k does the $\lim_{x \to 1} f(x)$ exist where f is defined by : **7+7**

$$f(x) = \begin{cases} 2kx + 3, & \text{if } x < 1 \\ 1 - kx^2, & \text{if } x > 1 \end{cases}$$

5. (a) Differentiate the following :

(i) $\tan^{-1} \dfrac{\sqrt{1+x^2} - 1}{x}$

(ii) $\sqrt{\dfrac{1+\cos x}{1-\cos x}}$

(b) Evaluate $\dfrac{dy}{dx}$ from the following : **7+7**

(i) $y = \tan\left[\log \sin^{-1} x\right]$

(ii) $x^y = y^x$

Subject Code—52853

B.C.A. EXAMINATION

(Main/Re-appear) (Batch 2017 Onwards)

(First Semester)

MATHEMATICAL FOUNDATIONS-I

BCA-115

Time : 3 *Hours* *Maximum Marks :* 70

Note : Q. No. **1** is compulsory. The students are required to attempt *Five* questions in all, selecting *one* question from each of Units I-IV and the compulsory question. All questions carry equal marks.

Compulsory Question

1. (a) Define equal sets. Is the following sets are equal ? **3**

$$A = \left\{x : x^2 = 16, x \in N\right\}; B = \{-4\}$$

(b) Evaluate the value of n if : **3**

$$^{n}P_4 : {}^{n-1}P_3 = 9:1$$

(c) Define discontinuity of First and Second kinds. **2**

(d) Find $\frac{dy}{dx}$, if $y = \log(\sin x)$. **2**

(e) Define geometrical meaning of a differential equation. **2**

(f) Solve the differential equation : **2**

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 0$$

Unit I

2. (a) Define finite and infinite sets, singleton and null sets. Which of the following sets are singleton or null or infinite sets :

(i) A = $\{x : 3 < x < 4, x \in N\}$

(ii) B = $\{x : x^2 = 9, x \in N\}$

(iii) C = $\{x : x$ is an even prime number$\}$

(iv) D = $\{x : x^2 + 2x + 1 = 0, x \in N\}$

(v) E = $\{x : x \in N$ and x is prime$\}$

(vi) F = $\{x : x \in Z$ and $x > -10\}$

(b) Prove that :

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(c) What do you mean by equivalence relation on a set ? Show that the relation R on the set A = {0, 1, 2, 3, ..., 11, 12}, given by R = $\{(a, b) : |a - b|$ is a multiple of 4$\}$, is an equivalence relation. **5+5+4**

3. (a) How many three digit odd numbers can be formed from the digits 1, 2, 3, 4, 5, 6 when repetition of digits is not allowed ?

(b) Out of 6 boys and 4 girls, a committee of 6 is to be formed. In how many ways can this be done if the committee contains (i) exactly 2 girls; (ii) at least two girls ?

(c) Define Boolean algebra along with examples. **5+5+4**

Unit III

6. (a) Form a differential equation of the family of curves represented by :

$$y = ax + bx^2$$

where a and b are arbitrary constants.

(b) Solve the differential equation :

$$\frac{dy}{dx} = y\left(e^x + 1\right)$$

(c) Solve : **4+5+5**

$$\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$$

7. (a) Solve :

$$\frac{dy}{dx} + 2y = 4x$$

(b) Solve : **7+7**

$$\left(x^4 - 2xy^2 + y^4\right)dx - \left(2x^2y - 4xy^3 + \sin y\right)dy = 0$$

Unit IV

8. (a) Solve :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = e^{4x}$$

(b) Solve : **7+7**

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin 2x$$

9. (a) Solve :

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} + 2y = 10\left(x + \frac{1}{x}\right)$$

(b) Solve : **7+7**

$$(x+a)^2\frac{d^2y}{dx^2} - 4(x+a)\frac{dy}{dx} + 6y = x$$