(b) Find the points on the curve $y = x^4 - 6x^3 + 13x^2 = 10x + 5$, where the tangents are parallel to the line $y = 2x$ and prove that two of the points have the same tangents. **7**

5. (a) Expand $\tan(x)$ by Maclaurin's theorem as far as x^5 and hence find the value of $\tan 46°30'$ up to four decimal places. **7**

(b) Show that the function :

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}; & (x,y) \neq (0,0) \\ 0; & (x,y) = (0,0) \end{cases}$$

is continuous at (0, 0). **7**

Unit III

6. (a) Find all the asymptotes to the curve :

$$(x+y)^2(x+y+2) - x - 9y + 8 = 0. \quad \textbf{7}$$

(b) Find the position and the nature of the double points on the curve $x^3 + x^2 + y^2 - x - 4y + 3 = 0$. **7**

Subject Code—52861

B.C.A. EXAMINATION

(Main/Reappear Batch 2017 Onwards)

(Third Semester)

MATHEMATICAL FOUNDATIONS–III

BCA-236

Time : 3 Hours *Maximum Marks :* 70

Note : The question paper contains four Units. Q. No. **1** is compulsory. The student selecting *one* question from each Unit and compulsory question. All questions carry equal marks.

1. (a) If $y = \left(\sin^{-1} x\right)^2$, prove that :

$$\left(1 - x^2\right) y_2 - x y_1 = 2. \quad \textbf{2}$$

(b) Evaluate $\lim_{x \to 0} \frac{x \cos x - \log(1+x)}{x^2}$. **2**

(c) Expand $\sin\left(e^x - 1\right)$ up to and including the term of x^4. **2**

(d) Define asymptotes and its types. **2**

(e) Define multiple points and its types with the help of diagram. **2**

(f) To prove that for the curve $p = f(r)$, $\rho = r - \frac{dr}{dp}$. **2**

(g) What are the special point of the curve $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$? **2**

Unit I

2. (a) Find the derivative $\frac{dy}{dx}$ of the parametrically defined function $x = t + 2\sin \pi t$, $y = 3t - \cos \pi t$ at $t = \frac{1}{2}$. **7**

(b) Find the nth derivative of $\frac{x^2}{(x-1)^3 (x-2)}$. **7**

3. (a) (i) If $y = \sin x + e^x$, then find out $\frac{d^3 x}{dy^3}$.

(ii) If $x = \ell \times p\left\{\tan^{-1}\left(\frac{y - x^2}{x^2}\right)\right\}$, then find out value of $\frac{dy}{dx}$. **8**

(b) If $y = \left[\log\left(x + \sqrt{1+x^2}\right)^2\right]$, prove that :

$$\left(1+x^2\right) y_{n+2} + (2n+1) x y_{n+1} + n^2 y_n = 0.$$ **6**

Unit II

4. (a) Find the equation of the tangent to the curve $x = \frac{t-1}{t+1}$, $y = \frac{t+1}{t-1}$ at the point $t = 2$. **7**

7. (a) Find the asymptotes of the curve :

$$r\cos\theta = a\cos 2\theta.$$ **7**

(b) Find the points of inflexion of the curve $y = (x-2)^6 (x-3)^5$. **7**

Unit IV

8. (a) Show that radius of curvature at any point $P(x, y)$ of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is given by $\rho^3 = 27axy$. **7**

(b) Find the center of curvature for the curve $x^3 + y^3 = 3axy$ at $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. **7**

9. (a) Trace the curve $x^3 + y^3 = 3axy$. **7**

(b) Trace the curve $r = a\sin 3\theta$. **7**