Roll No. .... Exam Code : D-23

Subject Code—60382

B. Sc. (Maths) EXAMINATION

(Batch 2022 Onwards) (Main/Re-appear)

(First Semester)

MATHEMATICS

CML-106 (Course I)

Algebra

Time : 3 Hours *Maximum Marks* : 80

Note : Question Paper contains nine questions in all, two questions in each Unit and *one* compulsory question (Q. No. 1) distributed in five parts. Attempt *Five* questions in all, selecting *one* question from each Unit and the compulsory question. All questions carry equal marks.

Compulsory Question

1. (a) If A is a symmetric matrix, show that kA is also a symmetric matrix. 2

(b) Show that no skew-symmetric matrix can be of the rank one. 2

(c) The transpose of a unitary matrix is unitary. Prove. 2

(d) For what value of k, the system of equations :

$x + 2y = 0,\ 3x + ky = 0$, has more than one solution. 2

(e) Find an equation whose one root is $2 - 3i$. 2

(f) Diminish the roots of the equation $x^3 + 2x^2 + 3x - 1 = 0$ by 1. 2

(g) Show that the equation $x^8 + 25x^3 + 2x - 1 = 0$ has at least six imaginary roots. 2

(h) Explain Descarte's rule of sign. 2

Unit I

2. (a) If A and B are square matrices of the same order and A is non-singular, then prove that $|A^{-1}BA| = |B|$. 8

(b) Find the condition under which the rank of the $\begin{bmatrix} 1 & 0 & 0 \\ 0 & p-2 & 2 \\ 0 & q-1 & p+2 \\ 0 & 0 & 3 \end{bmatrix}$ will be less than 3. Also find that rank. 8

3. (a) Find the minimal equation of matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7 \end{bmatrix}$ and show that A is non-derogatory. 8

(b) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$ and compute A^{-1}. 8

Unit II

4. (a) Solve that the only real values of λ for which the equations : 8

$$x + 2y + 3z = \lambda x$$
$$3x + y + 2z = \lambda y$$
$$2x + 3y + z = \lambda z$$

have a non-zero solution is 6. 8

(b) Reduce the quadratic form $x^2 - 2y^2 + 3z^2 - 4yz + 6zx$ to canonical form and find the rank, index and signature of the form. Also, find the equations of linear transformations. 8

5. (a) Solve the following system of equations completely : 8

$$2x - y + 3z = 3$$
$$x + 2y - z - 5w = 4$$
$$x + 3y - 2z - 7w = 5$$

(b) Prove that the product of two unitary matrices is unitary. 8

Unit III

6. (a) If b and c are real and $2-\sqrt{-3}$ is a root of the equation $x^3 + x^2 + bx + c = 0$, what are the other roots and what is the value of c ? 8

(b) If α, β, γ are the roots of the equation $ax^3 + 3bx + 3cx + d = 0$, from an equation whose roots are $\alpha + \beta$, $\beta + \gamma$, $\alpha + \gamma$. 8

7. (a) Find the condition that the sum of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$, is equal to zero. 8

(b) Find the equation squared difference of the roots of the equation $x^3 - 7x + 6 = 0$. 8

Unit IV

8. (a) Discuss the nature of the roots of the cubic $Z^3 + 3HZ + G = 0$. 8

(b) Solve the equation $x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$ by Ferrari's method. 8

(a) Solve by the method of resolution into quadratic factors :

$$x^4 - 2x^3 - 5x^2 + 10x - 3 = 0.$$ 8

(b) Show that the equation :

$$x^3 - qx + r = 0,$$

where q and r are essentially positive, has one negative root and that the other two roots are either imaginary or both positive. 8