Roll No. 22311232 0007
Hari

Exam Code : D-22

Subject Code—58375

B. Sc. EXAMINATION

(Main) (Batch 2022 Onwards)

(First Semester)

MATHEMATICS

CML-106, Course–I

Algebra

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Prove that every skew-symmeric matrix of odd order is singular matrix.

(b) Prove that O is latent root of a matrix iff A is singular.

(c) Show that transpose of a unitary matrix is unitary.

(d) If quadratic form X′AX is positive definite, then $|A| > 0$.

(e) If the roots of the equation $x^n - 1 = 0$ be $1, \alpha, \beta, \gamma, \ldots\ldots$, show that :

$$(1-\alpha)(1-\beta)(1-\gamma)\ldots\ldots = n.$$

(f) Show that if α be a root of $f(x) = 0$ repeated r times, then is a root of $f'(x) = 0$, repeated $r - 1$ times.

(g) Find an equation whose roots are equal in magnitude but opposite in sign to the roots of the equation :

$$x^5 + 11x^4 + 7x^3 - 16x^2 - 12x + 15 = 0 \cdot$$

(h) Apply Descarte's rule of sign to prove that all the roots of :

$$x^4 + 3x^3 + 5x^2 + 2x - 1 = 0$$

are real.

Unit I

2. (a) Show that every square matrix is uniquely expressible as the sum of a Hermitian and a Skew-Hermitian matrix.

(b) Using elementary operations find the inverse of the matrix $\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 5 & 2 & 3 \end{bmatrix}$.

3. (a) If the set $\{x_1, x_2, \ldots\ldots, x_p\}$ is linearly independent and the set $\{x_1, x_2, \ldots\ldots, x_p, y\}$ is linearly dependent, then show that y will be linear conbination of the vectors $x_1, x_2, \ldots\ldots, x_p$.

(b) If $A = \begin{bmatrix} 1 & \sqrt{2} \\ -\sqrt{2} & -1 \end{bmatrix}$, compute :

$$B = 3A - 2A^2 - A^3 - 5A^4 + A^5.$$

Unit II

4. (a) Solve the system of equations :

$$\lambda x+2y-2z=1;$$

$$4x+2\lambda y-z=2;$$

$$6x+6y+\lambda z=3,$$

considering specially the case when $\lambda=2$.

(b) Solve the system of equations :

$$n-2y+z-w=0,$$

$$x+y-2z+3w=0,$$

$$4x+y-5z+8w=0,$$

$$5x-7y+2z-w=0.$$

5. (a) Prove that the characteristic roots of a unitary matrix are of unit modulus.

(b) Express :

$$x_1y_1-2x_1y_3+3x_2y_1+x_2y_2-3x_2y_3-x_3y_1-x_3y_2-x_3y_3$$

into matrix form and transform into canonical form. Also find equation of transformation.

Unit III

6. (a) Show that the equation :

$$\frac{A^2}{x-a}+\frac{B^2}{x-b}+\frac{C^2}{x-c}+\ldots+\frac{H^2}{x-h}=K$$

has all real roots.

(b) Solve the equation :

$$x^4-6x^3+11x^2-18x+9=0,$$

given that the ratio of two roots is equal to the ratio of the other two.

7. (a) Find the condition that the roots of the cubic $x^3-px^2+qx+r=0$ may be in H.P. Hence solve the equation $6x^3-11x^2+6x-1=0$.

(b) Reduce the Cubic $2x^3-3x^2+6x-1=0$ to the form $Z^3+3HZ+G=0$, where H and G are integers.

Unit IV

8. (a) Solve the equation :

$$x^3+x^2-16x+20=0$$

by Cardan's method.

(b) Solve that the equation :

$$x^4-6x^3+8x^2+2x-1=0$$

by Descarte's method.

9. (a) Show that the equation :

$$x^3+x^2-2x-1=0$$

has three real roots.

(b) Show that for all value of constant α, the equation $3x^5+6x^2+10x+\alpha=0$ has at least two imaginary roots.