Roll No. 2331123200[redacted]58 Exam Code : D-23

Subject Code—60383

B. Sc. EXAMINATION

(Batch 2022 Onwards) (Main/Re-appear)

(First Semester)

MATHEMATICS

CML-107 (Course II)

Calculus

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

1. (a) Evaluate : 2

$$\lim_{n\to\infty}\frac{\sum n^2}{n^3}$$

(b) If $y=\left(\sin^{-1}x\right)^2$, then prove that : 2

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$$

P.T.O.

(c) Prove that a curve with constant curvature is always a circle. 2

(*d*) Prove that the point (0, 0) is a conjugate point on the curve : 2

$$x^4 + y^3 + 2x^2 + 3y^2 = 0$$

(e) Evaluate : 2

$$\int_0^{\pi} \sin^6 \frac{\theta}{2} \cos^8 \frac{\theta}{2} d\theta$$

(f) Find the *length* of the arc of the curve

$y = \log \sec x$ from $x = 0$ to $x = \frac{\pi}{3}$. 2

(g) Find the area bounded by $r^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$. 2

(h) Define surface of revolution along with a suitable example. 2

Unit I

2. (a) Show that the function : 8

$$f(x) = \begin{cases} x\left(\dfrac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}\right), & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

is continuous but not derivable at the point $x = 0$.

(b) Using Leibnitz's theorem, show that if $y = x^2 e^x$, then : 8

$$y_n = \frac{n(n-1)}{2} y_2 - n(n-2) y_1 + \frac{(n-1)(n-2)}{2} y$$

3. (a) Using Maclaurin's theorem, prove that :

$$\tan^{-1}\left(\frac{\sqrt{1+x^2} - 1}{x}\right) = \frac{1}{2}\left(x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots\ldots\right)$$

8

(b) Calculate the approximate value of $\sqrt{17}$ to four decimal places by taking the first three terms of a Taylor's series expansion. 8

Unit II

4. (a) Find all the asymptotes to the curve : 8

$$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$$

(b) Find a point on the parabola $y^2 = 8x$, at which the radius of curvature is $\frac{125}{16}$. 8

5. (a) Find the position and nature of the double points on the curve : 8

$$x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$$

(b) Find the range of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave or convex upwards. Also find the points of inflexion on the curve. 8

Unit III

6. (a) Trace the curve : 8

$$y^2(x^2 + y^2) + a^2(x^2 - y^2) = 0$$

(b) If $I_n = \int_0^{\pi/2} x \cos^n x \, dx$, then prove that : 8

$$I_n = \left(\frac{n-1}{n}\right) I_{n-2} - \frac{1}{n^2}$$

7. (a) Find the length of one arch of the cycloid : 8

$$x = a\,(\theta - \sin\theta),\ y = a\,(1 - \cos\theta)$$

(b) Connect the integral $\int x^m \left(a + bx^n\right)^p dx$ with $\int x^m \left(a + bx^n\right)^{p+1} dx$. 8

Unit IV

8. (a) Find the area of the smaller portion enclosed by the curves $y^2 = 8x$ and $x^2 + y^2 = 9$. 8

(b) Find the surface area of the right circular cone formed by the revolution of a right angled triangle about a side which contains the right angle. 8

9. (a) Find the area of a loop of the curver $r = a\cos 2\theta$ and hence find the total area of the curve. 8

(b) Find the volume of the solid formed by the revolution about the x-axis of the curve $y^2(a + x) = x^2(a - x)$. 8