Roll No. 22311232007
Hari

Exam Code : D-22

Subject Code—58376

B. Sc. EXAMINATION

(Main) (Batch 2022 Onwards)

(First Semester)

MATHEMATICS

CML-107, Course-II

Calculus

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) If a function f is continuous at a, then prove that $|f|$ is also continuous at a. **2**

(b) Find the nth derivative of $\log(ax+b)$. **2**

(c) Find the assymptotes parallel to the co-ordinate axes of the curve :

$$xy^3 - x^4 = a\left(x^2 + y^2\right).$$ 2

(d) Show that the curve $a^4y^2 = x^5(2a - x)$ has a single cusp of the first species at the origin. 2

(e) Derive reduction formula for :

$$\int x^m (\log x)^n \, dx.$$ 2

(f) Evaluate $\int_0^\infty \frac{dx}{\left(1+x^2\right)^n}$ by using reduction formula. 2

(g) Prove that the length of the arc of the curve $p = r\sin\beta$ between the limits r_1 and r_2 in $(r_1 - r_2)\sec\beta$. 2

(h) Derive the intrinsic equation of a curve from the Cartesian equation. 2

Section I

2\. (a) Let f be the continuous function defined on R by setting :

$$f(x)=\begin{cases} x^3 \sin\frac{1}{x}; & \text{if } x \neq 0 \\ 0; & \text{if } x = 0 \end{cases}.$$

Show that f is continuous on R but is not derivable at 0. **8**

(b) If $y=(\sin n^{-1}x)^2$, prove that :

$$(1+x^2)y_{n+2}+(2n+1)xy_{n+1}+n^2y_n=0,$$

also find $y_n(0)$. **8**

3. (a) Expand a^x by Maclaurin's theorem with Lagrange's form of remainder after n terms. **8**

(b) Obtain first four terms in the expansion of $\log\sin x$ in power of $(x-3)$. **8**

Section II

4. (a) Find the asymptotes of $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$. **8**

(b) Show that the radius of curvature for the catenary $y = c\cosh\frac{x}{c}$ is equal to the portion of the normal intercepted between the curve and the x-axis and that it varies as the square of the ordinate. 8

5. (a) If c_x and c_y be the chords of curvature parallel to co-ordinate axes at any point of the curve $y = e^{x/a}$, prove that :

$$\frac{1}{c_x^2} + \frac{1}{c_y^2} = \frac{1}{2ac_x}.$$ 8

(b) Show that the curve $y^3 = (x-a)^2(2x-a)$ has a single cusp of first species at $(a, 0)$. 8

Section III

6. (a) Trace the curve $r = a(1-\sin\theta)$. 8

(b) Find the length of the complete cycloid given by :

$$x = a(\theta + \sin\theta),\ y = a(1+\cos\theta).$$ 8

7. (a) Evaluate $\int_0^{\pi/2} \cos^n x \, dx$, where n is a +ve even and odd integer. 8

(b) Find the intrinsic equation of the Cardioid $r = a(1-\cos\theta)$. 8

Section IV

8. (a) Find the area of a loop of the curve $r = a\cos 2\theta$ and hence find the total area of the curve. 8

(b) Prove that the area common to the circles $r = a\sqrt{2}$ and $r = 2a\cos\theta$ is $a^2(\pi - 1)$. 8

9. (a) Show that the volume of the solid formed by the revolution of the curve $r = a + b\cos\theta \, (a > b)$ about the initial line is $\frac{4}{3}\pi a(a^2 + b^2)$. 8

(b) Find the surface of a sphere of radius a. 8

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