Section III

6. (a) Solve the equation :

$$x^6 - 4x^5 - 11x^4 + 40x^3 + 11x^2 - 4x - 1 = 0$$

given that one root is $\sqrt{2} + \sqrt{3}$. **4**

(b) Solve the equation :

$$x^4 - 6x^3 + 11x^2 - 18x + 9 = 0,$$

given that the ratio of two roots is equal to the ratio of other two. **4**

7. (a) Solve the equation :

$$15x^4 - 8x^3 - 14x^2 + 8x - 1 = 0,$$

given that the roots are in H. P. (Harmonic Progression). **4**

(b) Find the equation whose roots are the squered difference of the roots of the cubic $x^3 - 6x + 4\sqrt{2} = 0$. Hence prove that the given equation has multiple root. **4**

Roll No. ..... Exam Code : M-21

Subject Code—52458

B. Sc. EXAMINATION

(Reappear Batch 2017)

(First Semester)

MATHEMATICS

BM-111

Algebra

Time : 3 *Hours* *Maximum Marks :* 40

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

1. (a) If $|A| \neq 0$, prove that adjoint of A is non-singular. **2**

(b) Show that the diagonal elements of a skew-symmetric matrix are all zero. **1½**

(c) If A is Unitary and Hermitian matrix, then show that A is involutory. **1½**

(d) Solve the equation :

$$x^3 - 7x^2 + 14x - 8 = 0,$$

the roots being in G. P. **1½**

(e) Show that the equation $x^{2n} - 1 = 0$ has only two real roots. **1½**

Section I

2. (a) If A is a non-singular matrix of order n, prove that $\text{adj}(\text{adj}\,A) = |A|^{n-2} A$. **4**

(b) Find non-singular matrices P and Q such that PAQ is in the normal form, where

$$A = \begin{bmatrix} 3 & 1 & 2 & 1 \\ 1 & 4 & 6 & 1 \\ 2 & -3 & 1 & -2 \end{bmatrix}.$$ **4**

3. (a) If A is an n square matrix of rank $n-1$, show that $\text{adj}(A) \neq 0$. **3**

(b) Prove that constant term of characteristic polynomial i.e. $\det(\lambda I_n - A)$ is $(-1)^n \det(A)$. **5**

Section II

4. (a) Find the values of a and b for which the following system of linear-equations $2x + by - z = 3$, $5x + 7y + z = 7$ and $ax + y + 3z = a$ has an infinite number of solutions. **5**

(b) Show that the matrix $A = \begin{bmatrix} a+ic & -b+id \\ b+id & a-ic \end{bmatrix}$ is unitary iff $a^2 + b^2 + c^2 + d^2 = 1$. **3**

5. (a) If A is a real skew-symmetric matrix, such that $A^2 + I = \theta$, show that A is orthogonal and is of even order. **4**

(b) Prove that the quadratic form $-3x^2 - 4y^2 + 2z^2 - 2yz + 2x + 7xy$ is neither positive definite nor negative definite. **4**

Section IV

8. (a) Solve the equation $28x^3 - 9x^2 + 1 = 0$ by Cardan's method. **4**

(b) Solve the equation :

$$x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$$

by the method of resolution into quadratic factors. **4**

9. (a) Show that the equation :

$$x^7 + x^4 + 8x + k = 0$$

has at least four imagenary roots for all values of k. **3**

(b) Solve the equation :

$$x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$$

by Ferrari's method. **5**