(b) Find the range of values of x for which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards or downwards. Also find the points of inflexion on the curve. **4**

Unit III

6. (a) Trace the curve : **4**

$$r = a(1 - \sin\theta)$$

(b) If $I_n = \int_0^{\pi/2} \theta \sin^n \theta d\theta (n > 1)$, then prove that $I_n = \left(\frac{n-1}{n}\right) I_{n-2} + \frac{1}{n^2}$. Hence deduce that $I_5 = \frac{149}{225}$. **4**

7. (a) Find the whole length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. **4**

(b) Find the intrinsic equation of the cardioid $r = a(1 + \cos\theta)$ and hence show that $s^2 + 9p^2 = 16a^2$, where ρ is the radius of curvature and s is arc length. **4**

Roll No. ..... **Exam Code : M-21**

Subject Code—52459

B. Sc. EXAMINATION

(Reappear) (Batch 2017)

(First Semester)

MATHEMATICS

BM-112

Calculus

Time : 3 *Hours* *Maximum Marks :* 40

Note : Attempt *Five* questions in all including Q. No. **1** which is compulsory. Select *one* question from each Unit. All questions carry equal marks.

(Compulsory Question)

1. (a) Evaluate : **1**

$$\lim_{x \to 0} \frac{1 - \cos x}{\sin^2 x}$$

(b) Define removable discontinuity and infinite discontinuity. **1**

(c) Define node and cusp along with suitable examples. **2**

(d) Evaluate : **2**

$$\int_0^{3\pi/2} \cos^5\left(\frac{\theta}{3}\right) d\theta$$

(e) Define solid of revolution along with suitable example. **2**

Unit I

2. (a) Show that the function : **4**

$$f(x) = \left|x^2 - x + 1\right|$$

is continuous for every real number.

(b) Using Leibnitz's theorem, show that if $y = x^2 e^x$, then : **4**

$$y_n = \frac{n(n-1)}{2} y_2 - n(n-2) y_1 + \frac{(n-1)(n-2)}{(2)} y.$$

3. (a) Using Maclaurin's theorem, prove that : **4**

$$\log(1+\sin x) = x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + ...$$

(b) Using Taylor's theorem, expand $\sin(x+y)$ in powers of y. Hence find the value of $\sin 31^\circ$ upto four places of decimals. **4**

Unit II

4. (a) Find the asymptotes in the curve : **4**

$$x^3 + 4x^2y + 5xy^2 + 2y^3 + 2x^2 + 4xy + 2y^2 - x - 9y + 1 = 0$$

(b) Find the radius of curvature at any point for the curve : **4**

$$x = a(\cos t + t\sin t),\ y = a(\sin t - t\cos t)$$

5. (a) Prove that the point (1, 2) is a node on the curve $y(x-1)^2 = x(y-2)^2$. Also find the equations of the tangents to the curve at this point. **4**

Unit IV

8. (a) Find the area between the curve $x^2y^2 = a^2\left(y^2 - x^2\right)$ and its asymptote. **4**

(b) Find the surface area of the right circular cone formed by the revolution of a right angled triangle about a side which contains the right angle. **4**

9. (a) Find the area of a loop of the curve $x^5 + y^5 = 5ax^2y^2$. **4**

(b) Find the volume of the solid generated by the revolution of the curve $\left(a - x\right)y^2 = a^2x$ about its asymptote. **4**

Unit IV

8. (a) Find the area between the curve $x^2y^2 = a^2\left(y^2 - x^2\right)$ and its asymptote. **4**

(b) Find the surface area of the right circular cone formed by the revolution of a right angled triangle about a side which contains the right angle. **4**

9. (a) Find the area of a loop of the curve $x^5 + y^5 = 5ax^2y^2$. **4**

(b) Find the volume of the solid generated by the revolution of the curve $\left(a - x\right)y^2 = a^2x$ about its asymptote. **4**