Unit II

4. (a) Solve the following system of equations completely :

$$2x - y + 3z = 3$$
$$x + 2y - z - 5w = 4$$
$$x + 3y - 2z - 7w = 5$$

(b) Find the value of k such that the following system of equations has a non-trivial solution :

$$x + ky + 3z = 0$$
$$4x + 3y + kz = 0$$
$$2x + y + 2z = 0$$

8,8

5. (a) Prove that the absolute value of each characteristic root of unitary matrix is unity.

(b) Diagnolize the quadratic form $x^2 + 2y^2 - 7z^2 - 4xy + 8xz$. Also, find the rank, index, signature and equations of transformations. **8,8**

Roll No. .... Exam Code : J-21

Subject Code—52461

B. Sc. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(First Semester)

MATHEMATICS

CML-106

Algebra

Time : 3 Hours *Maximum Marks :* 80

Note : Attempt any *Five* questions. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Pove that the diagonal elements of a skew-symmetric matrix are all zero. **2**

(b) If $|A| \neq 0$, then prove that the : **2**

$$|A^{-1}| = |A|^{-1}$$

(c) If A and B are symmetric matrices, prove that AB is symmetric if and only if AB = BA. **2**

(d) Find p, if the vectors $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ p \end{bmatrix}$ are linearly dependent. **2**

(e) Prove that the determinant of an orthogonal matrix is ± 1. **2**

(f) Show that the equation $x^8 + 5x^3 + 2x - 3 = 0$ has at least six imaginary roots. **2**

(g) Given that -4 is a root of the equation $2x^3 + 6x^2 + 7x + 60 = 0$. Find the other roots. **2**

(h) Find an equation whose roots are equal in magnitude but opposite in sign to the roots of the equation $x^5 + 11x^4 + 7x^3 - 16x^2 - 12x + 15 = 0$. **2**

Unit I

2. (a) Every square matrix A can be expressed in one and only one way as P + iQ, where P + Q are Hermitian matrices.

(b) Find the non-singular matrices P and Q such that P.A.Q. is in normal form where

$$A = \begin{bmatrix} 2 & 2 & -6 \\ -1 & 2 & 2 \end{bmatrix}.$$ **8,8**

3. (a) Verify Cayley-Hamilton Theorem for the matrix A and compute A^{-1}, where :

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}.$$

(b) Find the characteristic vectors of the matrix :

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}.$$ **8,8**

Unit IV

8. (a) Solve the equation $x^3 + x^2 - 16x + 20 = 0$ by Cardon's method.

(b) Solve the equation $40x^4 + 42x^3 + 3x^2 - 1 = 0$ by Descartes' method. **8,8**

9. (a) Solve the equation $x^4 - 4x^3 - 4x^2 - 24x + 15 = 0$ by Ferrari's method.

(b) Solve the equation $x^4 - 10x^3 + 35x^2 - 50x + 24 = 0$ by Descartes' method.

8,8

Unit III

6. (a) Solve the equation $x^4 - 8x^3 + 23x^2 - 28x + 12 = 0$, given that the difference of two of the roots is equal to the difference of the other two.

(b) If the product of two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ is equal to the product of the other two, show that $r^2 = p^2s$. **8,8**

7. (a) The difference between two roots of the equation $x^3 - 13x^2 + 15x + 189 = 0$ is 2. Solve it by increasing the roots by 2.

(b) If α, β, γ are the roots of the equation $x^3 + 3hx + g = 0$; form an equation whose roots are $\frac{\alpha+1}{\beta+\gamma-\alpha}, \frac{\beta+1}{\gamma+\alpha-\beta}, \frac{\gamma+1}{\alpha+\beta-\gamma}$.

8,8