

5. (a) If  $C_o, C_p$  denote the chord of curvature of the cardioid  $r = a(1 + \cos \theta)$  along and perpendicular to the radius vector through any point-respectively, prove that : 8

$$3(C_o^2 + C_p^2) = 8aC_o$$

- (b) Find the position and nature of the double points of the curve : 8

$$2(x^3 + y^3) - 3(x^2 + y^2) + 12x - 4 = 0$$

#### Unit IV

6. (a) If  $I_m = \int_0^1 x^n (\log x)^m dx$  ( $n \geq 0$  and  $m$  is +ve integer), prove that :

$$I_m = -\frac{m}{n+1} I_{m-1} \text{ hence evaluate } I_3. \quad 8$$

- (b) If  $u_n = \int_0^{\pi/2} \theta \sin^n \theta d\theta$  ( $n > 1$ ); prove that :

$$u_n = \frac{n-1}{n} u_{n-2} + \frac{1}{n^2} \text{ and deduce that}$$

$$u_5 = \frac{149}{225}. \quad 8$$

## Subject Code—52462

### B.Sc. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(First Semester)

MATHEMATICS

CML-107

Calculus

Time : 3 Hours

Maximum Marks : 80

**Note :** Attempt any *Five* questions. Q. No. **1** is compulsory. All questions carry equal marks.

#### Unit I

1. (a) Evaluate : 2

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{\sin^2 x}$$

(b) Define asymptotes with example. 2

(c) If  $y = e^{m \sin^{-1} x}$ , prove that : 2

$$(1-x^2)y_2 - xy_1 = m^2 y$$

(d) What is the point of inflexion ? 2

(e) Evaluate : 2

$$\int_0^{\pi/2} \sin^5 x \cos^4 x dx$$

(f) Find the area bounded by  $y^2 = 4ax$  and its latus rectum. 2

(g) Define Pappu's Theorem for volume. 2

(h) Find the length of loop of the curve  $r = a(\theta^2 - 1)$ . 2

### Unit I

2. (a) Using  $\epsilon$ - $\delta$  definition, prove that : 8

$$f(x) = \begin{cases} x \cos \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous at  $x = 0$ .

(b) If  $y = \left[ \log \left( x + \sqrt{1+x^2} \right) \right]^2$ ; prove that : 8

$$(1+x^2)y_{n+2} + (2n+1)xy_{n+1} + n^2 y_n = 0$$

3. (a) If  $y = e^{m \cos^{-1} x}$ , show that : 8

$$(1+x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2 + m^2)y_n = 0$$

(b) State and prove Taylor's theorem with Lagrange's form of remainder after  $n$  terms. 8

### Unit III

4. (a) Find all the asymptotes to the curve : 8

$$(x+y)^2(x+y+2) - x - 9y + 2 = 0$$

(b) Find the radius of curvature for the ellipse : 8

$$p^2 = a^2 \cos^2 \psi + b^2 \sin^2 \psi$$

- (b) Show that the area of the surface generated by the revolution of an arc of the cycloid :

$$x = a(\theta - \sin \theta); y = a(1 - \cos \theta)$$

about the line  $y = 2a$  is  $\frac{32}{3}\pi a^2$ . **8**

7. (a) Show that the length of the curve : **8**

$$x^2(a^2 - x^2) = 8a^2y^2 \text{ is}$$

$$\pi a\sqrt{2}.$$

- (b) Find the entire length of the cardioid  $r = a(1 + \cos \theta)$  and show that the arc of upper half is bisected by  $\theta = \frac{\pi}{3}$ . **8**

### Unit V

8. (a) Find the area common to the parabola

$$y^2 = 4ax \text{ and } x^2 = 4ay. \quad \mathbf{8}$$

- (b) Find the area of curve  $x = a \cos^3 t,$

$$y = b \sin^3 t. \quad \mathbf{8}$$

9. (a) The circle  $x^2 + y^2 = a^2$  is revolved about the  $x$ -axis. Find the volume of the sphere so formed. **8**