Roll No. .... Exam Code : M-19

Subject Code—8655

B. Sc. EXAMINATION

(Batch 2018 Onwards)

(Second Semester)

MATHEMATICS

CML-207

Calculus and Geometry

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) For which value of λ the equation $x^2+2xy+(1+\lambda)y^2+2\lambda y-1=0$ represent a pair of intersecting lines ?

(b) State various order of contract when two conics intersect.

(c) Find the equation of conics confocal with

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1.$$

(d) Find the polar equation of a circle with centre (r_1, θ_1) and radius a.

(e) Find the centre and radius of sphere :

$$2x^2+2y^2+2z^2-2x+4y+2z-5=0$$

(f) Define great circle.

(g) Define right circular cylinder and its axis.

(h) Find the condition that the cone, $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$, may have three mutually perpendicular generators. **2×8=16**

Section I

2. (a) If $\vec{r}$ is any vector, show that $\vec{r}=(\vec{r}.\vec{a}')\vec{a}'+(\vec{r}.\vec{b}')\vec{b}+(\vec{r}.\vec{c}')\vec{c}$, where $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal vectors of $\vec{a}, \vec{b}, \vec{c}$. **8**

(b) If $\hat{r}$ is unit vector in the direction of $\hat{r}$, then prove that :

$$\hat{r} \times \frac{d\hat{r}}{dt} = \frac{1}{r^2} \hat{r} \times \frac{d\hat{r}}{dt}$$

where $|\hat{r}| = r$. 8

3. (a) Find the constants $\vec{a}, \vec{b}, \vec{c}$ show that the directional derivatives of $\psi(x, y, z) = axy^2 + byz + cz^2x^3$ at (1, 2, –1) has maximum magnitude 64 in a direction parallel to z-axis. 8

(b) Show that :

$$\nabla^2\left(\nabla . \frac{\vec{r}}{r^2}\right) = 2r^{-4}$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ with $|\vec{r}| = r$. 8

Section II

4. (a) Find the circulation of $\vec{f}$ around the curve C, where $\vec{f} = y\hat{i} + z\hat{j} + x\hat{k}$ and C in the circle $x^2 + y^2 = 1, z = 0$. 8

(b) If $\vec{f} = uxz\hat{i} - y^2\hat{j} + yz\hat{k}$; evaluate $\iint_S \vec{f}.\hat{n}\, dS$, where S in the surface S of the cube bounded by $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$. **8**

5. (a) Prove that :

$$\iiint_V \frac{1}{r^2} dV = \iint \frac{\vec{r}.\hat{n}}{r^2} dS,$$

where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\hat{r}| = r$. **8**

(b) Verify Green's theorem in plane for $\oint_C \left(xy + y^2\right)dx + x^2 dy$, where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **8**

Section III

6. (a) What conic does the equation

$$36x^2 + 24xy + 29y^2 - 72x + 126y + 81 = 0$$

represent ? Find its centre and the equation referred to the centre as origin. **8**

(b) Find the equation of director circle of the conic : **8**

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0.$$

7. (a) Find the equation of the parabola which touch the conic $x^2 + xy + y^2 - 2x - 2y + 1 = 0$ at the point where it is cut by the line $x + y + 1 = 0$. **8**

(b) Find the condition that the line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may be a tangent to the conic $\frac{l}{r} = 1 + e\cos\theta$. **8**

Section IV

8. (a) Tangent plane at any point of the sphere $x^2 + y^2 + z^2 = r^2$ meets the Axes in A, B, C. Show that the locus of the point of intersection of the planes drawn parallel to the co-ordinate plane through A, B, C in the sphere $x^{-2} + y^{-2} + z^{-2} = r^{-2}$. **8**

(b) Find the limiting points of the co-axial system of spheres : **8**

$$x^2+y^2+z^2+3x-3y+6=0$$
$$x^2+y^2+z^2-6y-6z+6=0$$

9. (a) Find the equation of the cone with vertex (1, 1, 1) and passing through the curve of intersection of $x^2+y^2+z^2=1$ and $x+y+z=1$. **8**

(b) Find the equation of the right circular cylinder, whose guiding circle is $x^2+y^2+z^2=9$, $x-y+z=3$. **8**