

Roll No. 223112320007
Hau

Exam Code : M-23

Subject Code—59288

B.Sc. (Maths.) EXAMINATION

(Main)

(Batch 2022 Onwards)

(Second Semester)

MATHEMATICS

CML-206, Core Course-III

Ordinary Differential Equations and Laplace
Transformation

Time : 3 Hours

Maximum Marks : 80

Note : Attempt *five* questions in all, selecting *one*
from each Unit. Q. No. 1 is compulsory.
Each question carries equal marks.

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P.T.O.

(Compulsory Question)

1. (a) What is necessary and sufficient condition for a Differential Equation $Mdx + Ndy = 0$ to be exact.

- (b) What is geometrical meaning of solution of Differential Equation :

$$\frac{dy}{dx} = -\frac{y}{x}$$

- (c) What is oblique trajectory ?

- (d) Calculate p -discriminant of Differential Equation :

$$y = px + \frac{a}{p}$$

- (e) Solve the Differential Equation :

$$\frac{d^2y}{dx^2} + 16y = 0.$$

- (f) Write Laplace transform of $\sinh at$.

- (g) What is Auxiliary Equation of Differential Equation :

$$x^2 \frac{d^2 y}{dx^2} - 2x \frac{dy}{dx} - 4y = x^2$$

- (h) Write inverse Laplace transform of :

$$\frac{1}{(s-a)^2 - b^2} \quad 8 \times 2 = 16$$

Unit I

2. (a) Find the value of α , if y^α is an integrating factor of the Differential Equation :

$$2xydx - (3x^2 - y^2)dy = 0$$

- (b) Solve the Differential Equation :

$$(xy^3 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$$

$$2 \times 8 = 16$$

3. (a) Solve the differential equation :

$$xp^2 - 2yp - ax = 0.$$

- (b) Solve and find the complete primitive and singular solution of the equation

$$3y = 2px - \frac{2p^2}{x}. \quad 2 \times 8 = 16$$

Unit II

4. (a) Solve the differential equation :

$$\frac{d^4 y}{dx^4} + a^4 y = 0$$

- (b) Solve the differential equation :

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} + 4y = e^x \cos x. \quad 2 \times 8 = 16$$

5. (a) Solve the differential equation :

$$\frac{d^4 y}{dx^4} - y = x \sin x.$$

- (b) Solve the differential equation :

$$(1+x)^2 \frac{d^2 y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$$

$$2 \times 8 = 16$$

Unit III

6. (a) Solve the differential equation :

$$\frac{d^2 y}{dx^2} - 2 \tan x \frac{dy}{dx} + 5y = (\sec x)e^x$$

- (b) Solve the differential equation by method of variation of parameters :

$$\frac{d^2 y}{dx^2} - 6 \frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}. \quad 2 \times 8 = 16$$

7. (a) Solve the simultaneous differential equations :

$$\frac{dx}{dt} - 7x + y = 0$$

$$\frac{dy}{dt} - 2x - 5y = 0$$

- (b) Solve the differential equation :

$$\frac{dx}{y} = \frac{dy}{-x} = \frac{dz}{2x - 3y} \quad 2 \times 8 = 16$$

Unit IV

8. (a) Find the Laplace transform of $te^{-4t} \sin 5t$.
 (b) Evaluate :

$$L \int_0^t \frac{e^t \sin t}{t} dt \quad 2 \times 8 = 16$$

9. (a) Find the inverse Laplace transform of the function $\frac{s}{(s^2 + 1)(s + 1)^2}$.

- (b) Solve the differential equation

$$t \frac{d^2 y}{dx^2} + 2 \frac{dy}{dx} - 3y = \sin t, \quad y = \frac{dy}{dt} = 0 \quad \text{where}$$

$$t = 0 \text{ by transform method.} \quad 2 \times 8 = 16$$

$$\frac{\partial m}{\partial y} = 3ny^2 + 1 \quad \frac{\partial n}{\partial n} = 4ny^2 + 2$$

$$\frac{\partial m}{\partial y} \neq \frac{\partial n}{\partial n}$$

Integrating factor $\frac{\frac{\partial n}{\partial n} - \frac{\partial m}{\partial y}}{m} = \frac{1}{y}$

$$\Rightarrow e^{\int \frac{1}{y} dy} = e^{\log y}$$

multiply by given equation