Subject Code—56761

B. Sc. (Maths) EXAMINATION

(Main & Re-appear) (Batch 2018 Onwards)

(Second Semester)

MATHEMATICS

CML-206

Ordinary Differential Equations and Laplace Transformation

Time : 3 Hours *Maximum Marks : 80*

Note : Q. No. **1** is compulsory. Out of remaining eight questions (divided in 4 Units), attempt *four* questions by selecting either *one* question from each Unit or by selecting *two* questions from any one Unit and remaining *two* questions from two different Units out of remaining three Units.

Compulsory Question

1. (i) Give the geometrical meaning of the solution of the differential equation : 2

$$\frac{dy}{dx} = -\frac{x}{y}$$

(ii) Define exact differential equation. 2

(iii) Solve : 2

$$\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 25y = 0$$

(iv) Define orthogonal trajectory with the help of an example. 2

(v) Obtain the singular solution of the differential equation : 2

$$y = px + \frac{a}{p}$$

where $p = \frac{dy}{dx}$.

(vi) What is the condition which is sufficient for existence of Laplace transform of a function ? 2

(vii) Find Laplace inverse transform of $\frac{1}{s^2-3}$. 2

(viii) Find Laplace transform of t^2e^{3t}. 2

Unit I

2. (a) Solve the equation : 8

$$\left(3x^2+4xy\right)dx+\left(2x^2+2x\right)dy=0$$ +9)

(b) Find A such that the differential equation : 8

$$\left(x^2+3xy\right)dx+\left(Ax^2+4y\right)dy=0$$

is exact. Solve the resulting exact equation.

3. (a) Find the complete primitive and singular solution of : 8

$$4p^2(x-2)=1$$

(b) Solve the differential equation : 8

$$xp+y=x^4p^2$$

Unit II

4. (a) Find the orthogonal trajectories of the family of ellipses having centre at the origin, a focus at the point (C, 0) and semimajor axis of length 2C. 8

(b) Find the general solution of : 8

$$\frac{d^4y}{dx^4}-4\frac{d^3y}{dx^3}+14\frac{d^2y}{dx^2}-20\frac{dy}{dx}+25y=0$$

5. (a) Solve : 8

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=4\sin(\ln x)$$

(b) Solve : 8

$$\frac{d^3y}{dx^3}-6\frac{d^2y}{dx^2}+12\frac{dy}{dx}-8y=xe^{2x}+x^2e^{3x}$$

Unit III

6. By using the method of variation of parameters, solve the differential equation : 16

$$\left(x^2+1\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=6\left(x^2+1\right)^2$$

7. (a) Solve the system of equations : 8

$$2\frac{dx}{dt}-2\frac{dy}{dt}-3x=t$$

$$2\frac{dx}{dt}+2\frac{dy}{dt}+3x+8y=2$$

(b) Solve : 8

$$\frac{dx}{z(x+y)}=\frac{dy}{z(x-y)}=\frac{dz}{x^2+y^2}$$

Unit IV

8. (a) State and prove convolution theorem of Laplace transform. 8

(b) Evaluate : 8

$$\int_0^{\infty} t^3 e^{-t} \sin t \, dt$$

9. Solve the differential equation : 16

$$t\frac{d^2y}{dt^2}+\frac{dy}{dt}+4ty=0$$

$$\left.\frac{dy}{dt}\right|_{t=0}=0,\ y(0)=3$$

Using method of Laplace transform.