Roll No. 22311232007 Hari

Exam Code : M-23

# Subject Code—59289

## B. Sc. EXAMINATION

(Main) (Batch 2022 Onwards)

(Second Semester)

MATHEMATICS

CML-207, Core Course–IV

Vector Calculus and Geometry

*Time* : 3 *Hours*

*Maximum Marks* : 80

**Note** : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

**1.** (a) Show that :

$$\left(\vec{a}\times\vec{b}\right)\cdot\left(\vec{b}\times\vec{c}\right)\times\left(\vec{c}\times\vec{a}\right)=\left(\vec{a}\cdot\vec{b}\times\vec{c}\right)^2.$$

(b) If $\vec{f}$ and $\vec{g}$ are irrotational, prove that $\vec{f}\times\vec{g}$ is solenoidal.

(c) Evaluate :

$$\iint_S (x\,dy\,dz + y\,dz dx + z\,dx dy),$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

(d) Evaluate :

$$\int_3^4 \vec{r} \cdot \frac{d\vec{r}}{dt} dt,$$

where $\vec{r}(3) = 3\hat{i} + 2\hat{j} - \hat{k}$

and $\vec{r}(4) = 2\hat{i} - 5\hat{j} + \hat{k}$.

(e) Show that all the conics which pass through the intersection of two rectangular hyperbolas are themselves rectangular hyperbolas.

(f) Find the equation of the director circle of the conic :

$11x^2 + 24xy + 4y^2 - 2x + 16y + 11 = 0$.

(g) One end of the diameter of the sphere $x^2 + y^2 + z^2 - 3x - 2y + 2z - 15 = 0$ is at the point (–1, 4, 3).

Find the co-ordinates of the other end.

2.

(h) Find the equation of the quadric cylinder with generators parallel to $x$-axis and passing through the curve $ax^2+by^2+cz^2=1$, $lx+my+nz=p$.

**8×2=16**

## Unit I

**2.** (a) If :

$$\vec{a}=\sin\theta\,\hat{i}+\cos\theta\,\hat{j}+\theta\,\hat{k},$$

$$\vec{b}=\cos\theta\,\hat{i}-\sin\theta\,\hat{j}-3\hat{k},$$

$$\vec{c}=2\hat{i}+3\hat{j}-3\hat{k},$$

find :

$$\frac{d}{d\theta}\left\{\vec{a}\times\left(\vec{b}\times\vec{c}\right)\right\} \text{ at } \theta=\frac{\pi}{2}.$$

(b) If :

$$u=x+y+z,$$

$$v=x^2+y^2+z^2,$$

$$w=yz+zx+xy;$$

prove that :

$$(\operatorname{grad} u)\cdot\left[(\operatorname{grad} v)\times(\operatorname{grad} w)\right]=0.$$ **16**

3. (a) Find the constants $a$, $b$, $c$ so that the directional derivative of :

$$\phi(x, y, z) = axy^2 + byz + cz^2x^3,$$

at $(1, 2, -1)$ has a maximum of magnitude 64 in a direction parallel to $z$-axis.

(b) Find :

$$\vec{f} \times (\nabla \times \vec{g}) \text{ and } (\vec{f} \times \nabla) \times \vec{g}$$

at the point (1, –1, 2), where :

$$\vec{f} = xz^2\hat{i} + 2y\hat{j} - 13xz\hat{k},$$

$$\vec{g} = 3xz\hat{i} + 12yz\hat{j} - 8z^2\hat{k}.$$ 16

## Unit II

4. (a) Verify Stokes' theorem for $\vec{f} = x^2\hat{i} + xy\hat{j}$, integrated round the square in the plane $z = 0$ bounded by the lines $x = 0$, $y = 0$, $x = a$, $y = a$.

(b) Find the circulation of $\vec{f}$ round the curve C, when $\vec{f} = e^x \sin y \hat{i} + e^x \cos y \hat{j}$ and C is the rectangle whose vertices are $(0, 0), (1, 0), \left(1, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$. **16**

**5.** (a) Evaluate :

$$\iint_S \vec{f} \cdot \hat{n}\, dS,$$

where $\vec{f} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$, $z = 3$.

(b) Verify Green's theorem in plane for $\oint_C \left(x^2 - xy^3\right)dx + \left(y^2 - 2xy\right)dy$, where C is the square with vertices (0, 0), (2, 0), (2, 2), (0, 2). **16**

## Unit III

**6.** Trace the conic :

$$x^2 - 3xy + y^2 + 10x - 10y + 21 = 0.$$ **16**

7. (a) Prove that if :

$$ax^2+2hxy+by^2+2gx+2fy+c=0$$

is a parabola, then its latus rectum is

$$2\left[-\frac{\Delta}{(a+b)^3}\right]^{\frac{1}{2}}.$$

(b) Prove that the equation of the circle having double contact with the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, at the ends of latus rectum is :

$$x^2+y^2-2ae^3x+a^2\left(1-e^2-e^4\right).$$ **16**

## Unit IV

8. (a) Find the equations of the circle, circumscribing the triangle formed by the three points $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$ and obtain the co-ordinates of the centre of the circle.

(b) Show that :

$$33x^2 + 13y^2 - 95z^2 - 144yz - 96zx - 48xy = 0$$

represents a right circular cone whose axis is the line $3x = 2y = z$. Find the semi-vertical angle. **16**

**9.** (a) The point $(-1, 2, 1)$ is the limiting point of co-axial system of spheres in which

$$x^2 + y^2 + z^2 + 3x - 3y + 6 = 0$$

is a member. Find the equation of the radical plane of this system and the co-ordinates of the other limiting points.

(b) Obtain the equation of the right circular cylinder described on the circle through the three points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ as guiding circle. **16**