Subject Code—56762

B. Sc. EXAMINATION

(Batch 2018 Onwards)

(Main & Re-appear)

(Second Semester)

MATHEMATICS

CML-207

Vector Calculus and Geometry

Time : 3 Hours *Maximum Marks : 70*

Note : Q. No. 1 is compulsory. Out of remaining eight questions (divided in 4 Units), attempt *four* questions by selecting either *one* question from each Unit or by selecting *two* questions from any one Unit and remaining *two* questions from two different Units out of remaining three Units.

(Compulsory Question)

1. (a) Find :

$$\frac{d\vec{r}}{dt} \text{ and } \frac{d^2\vec{r}}{dt^2},$$

where

$$\vec{r} = (t+1)\,\hat{i} + (t^2+t+1)\,\hat{j} + (t^3+t^2+t+1)\,\hat{k}$$

(b) Prove that :

$$\nabla\phi.d\vec{r} = d\phi.$$

(c) If :

$$\int_1^2 \vec{a}.\left(\vec{b}\times\vec{c}\right)$$

when

$$\vec{a} = t\hat{i} - 3\hat{j} + 2t\hat{k},$$

$$\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$$

$$\vec{c} = 3\hat{i} + t\hat{j} + \hat{k}$$

(d) If S is any closed surface enclosing a volume V and $\vec{f} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then show that $\iint_S \vec{f}.\hat{n}\, dS = 6V$.

(e) What conic does the equation :

$$6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$$

represent ?

(f) If S = 0, represent a conic, T = 0 a tangent to it and u = 0 any other line, then interpret S + λT = 0.

(g) Find the centre and radius of the sphere given by :

$$x^2 + y^2 + z^2 + 2x - 4y + 6z - 2 = 0.$$

(h) Find the equation to the cone, whose vertex is at the origin and which passes through the curves :

$$\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{1} = 1, x + y + z = 1.$$

Unit I

2. (a) Given $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} + 3\hat{j} - \hat{k}$. Find the reciprocal triads $\vec{a}', \vec{b}', \vec{c}'$ and verify that $\left[\vec{a}\,\vec{b}\,\vec{c}\right]\left[\vec{a}', \vec{b}', \vec{c}'\right] = 1$.

(b) A particle moves along the curve :

$$x = t^3 + 1, y = t^2, z = 2t + 5,$$

where t is time. Find the component of velocity and acceleration at $t = 1$ in the direction of $\hat{i} + \hat{j} + 3\hat{k}$.

3. (a) Find the directional derivative of :

$$f(x, y, z) = xy + yz + zx$$

in the direction of the vector $2\hat{i} + 3\hat{j} + 6\hat{k}$ at the point (3, 1, 2).

(b) Evaluate :

$$\nabla . (\vec{r} \times \vec{a}),$$

where $\vec{a}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

Unit II

4. (a) Find the circulation of $\vec{f}$ around the curve C, where $\vec{f} = y\hat{i} + z\hat{j} + x\hat{k}$ and C is the circle $x^2 + y^2 = 1, z = 0$.

(b) Evaluate :

$$\iint_S \vec{f}.\hat{n}\, dS$$

where $\vec{f} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ which lies in the first quadrant.

5. (a) Evaluate :

$$\iint_S \vec{f}.\hat{n}\, dS$$

with the help of Gauss theorem for $f = 6z\hat{i} + (2x + y)\hat{j} - x\hat{k}$ taken over the region S bounded by the surface of the cylinder $x^2 + y^2 = 9$ included between $x = 0$, $y = 0$, $z = 0$ and $y = 8$.

(b) State and prove Green's Theorem.

Unit III

6. (a) Find the length of the axes, the eccentricity and the equations of the axes of the conic $5x^2 + 6xy + 5y^2 + 4x + 12y - 4 = 0$.

(b) Trace the conic :

$$8x^2 - 4xy + 5y^2 - 16x -$$

$$14y + 17 = 0$$

7. (a) Find the locus of mid-point of the chords of the confocal :

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1,$$

which touch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(b) Show that the locus of the point of intersection of two tangents to the parabolas :

$$\frac{l}{r} = 1 + \cos\theta,$$

which cut one another at a constant angle α is :

$$\frac{1}{r} = \cos\alpha + \cos\theta.$$

Unit IV

8. (a) A plane ABC, whose equation is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B, C. Find the equations to determine the circumcircle of the triangle ABC and obtain the co-ordinates of its centre.

   (b) Find the equations of the spheres which pass through the point (4, 1, 0), (2, – 3, 4), (1, 0, 0) and touch the plane $2x+2y-z=11$.

9. (a) Find the equation of the cone with vertex at (2, 3, 1) and passing through the curve of intersection of :

$$x^2+y^2+z^2-2x+4y-6z+7=0$$

and

$$x+2y+2z=5.$$

   (b) Find the equation of the right circular cylinder of radius 2 and axis as the line,

$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}.$$