Roll No. 22311222007 Hari

Exam Code : M-24

Subject Code—61373

B. Sc. EXAMINATION

(Main/Re-appear)

(Batch 2022 Onwards)

(Second Semester)

MATHEMATICS

CML-207, Core Course-IV

Vector Calculus and Geometry

Time : 3 Hours *Maximum Marks : 80*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Find the value of λ so that the following vectors are coplanar $\vec{a} = 2\hat{i} - 7\hat{j} + \lambda\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$ and $\vec{c} = 3\hat{i} - 5\hat{j} + 2\hat{k}$. 2

(b) If $\phi(x, y, z) = x^2y + y^2x + z^2$, find $\nabla\phi$ at the point (1, 2, 3). 2

(c) If $\vec{f} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, evaluate $\int_C \vec{f}.d\vec{r}$, where c is the line joining the point (0, 0, 0) to (1, 1, 1). 2

(d) Write the statement of Gauss's Divergence theorem. 2

(e) Find the equation of the tangent and normal to the conic $x^2 + 2xy - y^2 + 2x + 4y + 1 = 0$ at the point (– 2, 1). 2

(f) Find the equation of director circle of the conic $14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0$. 2

(g) Find the centre and radius of the sphere given by : 2

$$x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0$$

(h) Find the equation of the cone, whose vertex is at the origin and which passes through the curves $\frac{x^2}{4}+\frac{y^2}{9}+\frac{z^2}{1}=1$, $x+y+z=1$. 2

Unit I

2. (a) If $\vec{a}, \vec{b}, \vec{c}$ be three unit vectors such that $\vec{a}\times(\vec{b}\times\vec{c})=\frac{1}{2}\vec{b}$; find the angles which $\vec{a}$ makes with $\vec{b}$ and $\vec{c}$, where $\vec{b}$ and $\vec{c}$ are non-parallel. 8

(b) If $\vec{a}=\sin\theta\,\hat{i}+\cos\theta\,\hat{j}+\theta\hat{k}$; $\vec{b}=\cos\theta\,\hat{i}-\sin\theta\,\hat{j}-3\hat{k}$ and $\vec{c}=2\hat{i}+3\hat{j}-\hat{k}$; find $\frac{d}{d\theta}\left[\vec{a}\times(\vec{b}\times\vec{c})\right]$ at $\theta=0$. 8

3. (a) Find the directional derivative of $\phi=3y^2+yz^3$ at the point (2, –1, 1) in the direction of normal to the surface $x\log z-y^2+4=0$ at (– 1, 2, 1). 8

(b) Show that $r^n \vec{r}$ is irrotational where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $|\vec{r}| = r$. 8

Unit II

4. (a) Find the work done in moving a particle once round a circle C in the xy plane, if the circle has centre at the origin and radius 2 and if the force field is given as : 8

$$\vec{f} = (2x - y + 2z)\hat{i} + (x + y - z)\hat{j} + (3x - 2y - 5z)\hat{k}$$

(b) Evaluate : 8

$$\iint_S \vec{f}.\hat{n}dS,$$

where $\vec{f} = z\hat{i} + x\hat{j} + 3y^2 z\hat{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between $z = 0$ and $z = 5$.

5. (a) Apply Gauss theorem to show that

$$\iint_S [(x^3 - yz)\hat{i} - 2x^2 y\hat{j} + 2\hat{k}].\hat{n}\,dS = \frac{1}{3}a^5$$

where S denotes the surface of the cube bounded by the planes,

$x = 0,\ x = a,\ y = 0,\ y = a,\ z = 0,\ z = a.$ 8

(b) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$, where C is closed curve of the region bounded by $y = x$ and $y = x^2$. 8

Unit III

6. (a) What conic does the equation $36x^2 + 24xy + 29y^2 - 72x + 126y + 81 = 0$ represent ? Find its centre and the equation referred to the centre as origin. 8

(b) Trace the conic $9x^2 + 24xy + 16y^2 - 2x + 14y + 1 = 0$. 8

7. (a) Find the length of the axes, the eccentricity and the equations of the axes of the conic $5x^2 + 6xy + 5y^2 + 4x + 12y - 4 = 0$. 8

(b) Prove that the conics $x^2 + 3y^2 - 1 = 0$ and $2x^2 + 12xy + 39y^2 - 2x - 12y = 0$ have double contact with each other. Find the co-ordinates of the points the intersection of the tangents at the two points of contact. 8

Unit IV

8. (a) Show that the spheres $x^2 + y^2 + z^2 = 25$ and $x^2 + y^2 + z^2 - 18x - 24y - 40z + 225 = 0$ touch and find the co-ordinates of their common point. 8

(b) Find the equation of the rights circular cone whose vertex is at the origin, axis

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the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$, and which has a vertical angle of $60°$. 8

9. (a) Obtain the equation of the sphere having the circle :

$$x^2+y^2+z^2+7y-2z+2=0, \quad 2x+3y+4z-8=0$$

as a great circle. 8

(b) Find the equation of the right circular cylinder of radius 2 and axis as the line,

$$\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}.$$ 8