Roll No. .... Exam Code : D-24

Subject Code—62347

B. Sc. EXAMINATION

(Main/Re-appear)

(Batch 2022 Onwards)

(Third Semester)

MATHEMATICS

CML-306

Advanced Calculus

Course-V

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Define Discontinuity of second kind of a function at a point with example. 2

(b) Evaluate : **2**

$$\lim_{x \to \infty} \frac{\log x}{x}$$

(c) Show that : **2**

$$\lim_{(x,y)\to(0,0)} f(x,y) = 0$$

where

$$f(x,y) = x^2 + y^2$$

(d) Find the first order partial derivatives of $\log(x^2 + y^2)$. **2**

(e) Show that f_{xy} is not continuous at (0, 0) where : **2**

$$f(x,y) = \begin{cases} \frac{1}{4}(x^2 + y^2)\log(x^2 + y^2) & , \quad (x,y) \neq (0,0) \\ 0 & , \quad (x,y) = (0,0) \end{cases}$$

(f) If $u = x(1-y)$ and $v = xy$ then find $\frac{\partial(u,v)}{\partial(x,y)}$. **2**

(g) To show that : 2

$$\beta(m,n)=\beta(n,m)$$

(h) Evaluate the integral : 2

$$\int_0^1\int_0^x\int_0^{1+x+y} f(x,y,z)\,dx\,dy\,dz$$

where $f(x,y,z)=1$

Unit I

2. (a) Prove that the function $f(x)=2x^2+3x-4$ is uniformly continuous on $[-2, 2]$. 8

(b) Verify Lagrange's mean value theorem for the function $f(x)=\sqrt{25-x^2}$ in $[-3, 4]$. 8

3. (a) Show that for every value of x,

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \ldots\ldots$$ 8

$$(-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + (-1)^n \frac{x^{2n}}{(2n)!} \sin\theta x, \quad 0 < \theta < 1.$$

(b) Show that : 8

$$\lim_{x \to 0} \frac{(1+x)^{1/x} - e + \frac{ex}{2} - \frac{11}{24}ex^2}{x^3} = -\frac{7e}{16}$$

Unit II

4. ~~(a)~~ Let $f : \mathrm{R}^2 \to \mathrm{R}$ be defined as : 8

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + y^2} & , \quad (x, y) \neq (0,0) \\ 0 & , \quad (x, y) = (0,0) \end{cases}$$

Show that :

$$\lim_{(x,y) \to (0,0)} f(x, y)$$

does not exist.

(b) If $u = \sin^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$,

prove that : **8**

(i) $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$

(ii) $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$$

5. (a) Expand $e^x \cos y$ in terms of x and y as far as terms of third degree. **8**

(b) Show that at the point of contact, where the curves $f(x, y) = 0$ and $\phi(x, y) = 0$ touch, $\dfrac{\partial f}{\partial x}\dfrac{\partial \phi}{\partial y} - \dfrac{\partial f}{\partial y}\dfrac{\partial \phi}{\partial x} = 0$. **8**

Unit III

6\. (a) Show that the function : 8

$$f(x,y)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

is continuous and possesses first order partial derivatives at origin.

(b) Find the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. 8

7\. (a) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is cube. 8

(b) If f_x, f_y, f_{yx} all exist in the neighbourhood of point (a, b) and f_{yx} is continuous at the point (a, b), then $f_{xy}(a, b)$ also exist at (a, b) and $f_{xy}(ab) = f_{yx}(a, b)$. **8**

Unit IV

8. (a) If $u = x^2 - 2y$, $v = x + y + z$, $w = x - 2y + 3z$,

show that : **8**

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = 10x + 4.$$

(b) Show that : **8**

$$\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(a+bx)^{m+n}} dx = \frac{1}{(a+b)^m . a^n} B(m, n)$$

9. (a) Evaluate : **8**

$\iiint (x + y + z)\, dxdydz$ over the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$.

(b) Using Dirichlet's theorem, find the volume bounded by the surface : **8**

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^4}{c^4}=1$$