Roll No. 223112320007
Hari

Exam Code : D-23

Subject Code—60385

B. Sc. EXAMINATION

(Main) (Batch 2022 Onwards)

(Third Semester)

MATHEMATICS

CML-306, Course–V

Advanced Calculus

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Define removable discontinuity of a function at a point with example. **2**

(b) Evaluate $\lim\limits_{x \to a^+} \dfrac{\log(x-a)}{\log(e^x - e^a)}$ by L'Hospital's Rule. **2**

(c) Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x + y$, show that :

$$\lim_{(x, y) \to \left(0, \frac{1}{2}\right)} f(x, y) = \frac{1}{2}.$$ 2

(d) $z = \dfrac{1}{\sqrt{x^2 + y^2}}$; then find $\dfrac{\partial z}{\partial x}, \dfrac{\partial z}{\partial y}$. 2

(e) Let :

$$f(x, y) = \begin{cases} \frac{1}{4}\left(x^2 + y^2\right)\log\left(x^2 + y^2\right), & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}.$$

Show that f_{yx} is not continuous at (0, 0). 2

(f) If $u = x + y$, $v = \dfrac{y}{x + y}$, find $\dfrac{\partial(u, v)}{\partial(u, y)}$. 2

(g) Evaluate :

$$\int_0^1 \int_0^1 \int_0^1 e^{x+y+z}\, dx\, dy\, dz.$$ 2

(h) Show that $B(m, n) = B(n, m)$. 2

Unit I

2. (a) Show that the function f defined by :

$$f(x)=2x^2-3x+5$$

is uniformly continuous on [– 2, 2]. 8

(b) Use Lagrange's mean value theorem to determine a point on the curve $y=\sqrt{x^2-y}$ defined in [2, 4], where the tangent is parallel to the chord joining the end point of the curve. 8

3. (a) Show by means of Maclaurin's expansion that :

$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}....(-1)^n\frac{x^{2n}}{(2n)!}+(-1)^{n+1}$$

$$\frac{x^{2n+1}}{(2n+1)!}\sin(\theta x).\ 8$$

(b) Show that :

$$\lim_{x\to 0^+}\frac{(1+x)^{1/x}-e+\frac{ex}{2}}{x^2}=\frac{11e}{24}.\qquad 8$$

Unit II

4. (a) Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x, y) = \frac{x-y}{x+y}, \; x \neq 0, \; y \neq 0.$$ Show that

$$\lim_{(x, y) \to (0, 0)} f(x, y)$$ does not exist. 8

(b) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that : 8

(i) $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$$

(ii) $$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u\left(1 - 4\sin^2 u\right).$$

5. (a) Expand $e^x \sin y$ in powers of x and y as far as terms of third degree. 8

(b) If A, B, C are the angles of a triangle such that $\sin^2 A + \sin^2 B + \sin^2 C$ = constant, prove that :

$$\frac{dA}{dB} = \frac{\tan C - \tan B}{\tan A - \tan C}. \qquad 8$$

Unit III

6. (a) For function :

$$f(x, y) = \begin{cases} \dfrac{x^2 y}{x^4 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases},$$

show that the partial derivatives f_x, f_y exist everywhere in the region $-1 \leq x \leq 1$, $-1 \leq y \leq 1$. 8

(b) Find the maximum and minimum distance of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$ using Lagrange's method of multipliers. 8

7. (a) A rectangular box without top is to have volume 32 cubic feet. Find the dimensions of the box requiring least material for its construction. 8

(b) If $f : R^2 \rightarrow R$ be a function such that both f_x and f_y are differentiable at a point (a, b), then $f_{xy}(ab) = f_{yx}(a, b)$. 8

Unit IV

8. (a) If :

$$u = xyz, \quad v = xy + yz + zx,$$

$$w = x + y + z,$$

then show that :

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = (x-y)(y-z)(z-x).$$ 8

(b) Show that $B(m, n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$, $m > 0$, $n > 0$. 8

9. (a) Evaluate :

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} xyz \, dx \, dy \, dz \, . \qquad 8$$

(b) Using Dirichlet's integral, show that the volume of ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{4}{3}\pi abc$. 8