Roll No. 2131123 00255 **Exam Code : D-22**

Subject Code—58372

B. Sc. EXAMINATION

(Main/Re-appear) (Batch 2018 Onwards)

(Third Semester)

MATHEMATICS

CML-306 (Course V)

Advanced Calculus

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory.

Compulsory Question

1. (a) Write different kinds of discontinuity of a function. 2

(b) Differentiate $\sqrt{1+x^2}$ w.r.t. x. 2

(c) Evaluate : 2

$$\lim_{x \to b} \frac{x^b - b^x}{x^x - b^b}$$

(d) Let $f : R^2 \to R$ be defined as

$$f(x, y) = \frac{x - y}{x + y}, x \neq 0, y \neq 0.$$

Show that :

$$\lim_{(x,y) \to (0,0)} f(x, y)$$

does not exist. 2

(e) If $z = x^y + y^x$, find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$. 2

(f) Prove that every differentiable function is continuous. 2

(g) If $u = \log x + e^y + xyz$ and

$v = e^x + \log y + xyz$, then find $\frac{\partial(u, v)}{\partial(y, z)}$. 2

(h) Show that : 2

$$\int_{-\infty}^{0} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}.$$

Unit I

2. (a) Show that every function defined and continuous on a closed interval attains its bounds in that interval. 8

(b) State and prove Rolle's theorem. 8

3. (a) Show that : 8

$$\log(x+h) = \log x + \frac{h}{x} - \frac{h^2}{2x^2} + \ldots\ldots(-1)^{n-1} \frac{h^n}{n(x + \theta h)^n}$$

(b) Show that : 8

$$\lim_{x \to 0^+} \frac{(1+x)^{1/x} - e + \frac{ex}{2}}{x^2} = \frac{11e}{24}$$

Unit II

4. (a) Show that the function $f : R^2 \to R$ defined by : 8

$$f(x, y) = \begin{cases} xy\frac{x^2 - y^2}{x^2 + y^2}; & (x, y) \neq (0, 0) \\ 0; & \text{otherwise} \end{cases}$$

is continuous at (0, 0).

(b) If $u = x^y + y^x$, prove that : 8

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$$

5. (a) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}$, prove that : 8

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \sin 2u\left(1 - y\sin^2 u\right)$$

(b) Show that at point of contact, where the curves $f(x, y) = 0$ and $\phi(x, y) = 0$ touch : 8

$$\frac{\partial f}{\partial x}\frac{\partial \phi}{\partial y} - \frac{\partial f}{\partial y}.\frac{\partial \phi}{\partial x} = 0$$

Unit III

6. Show that the function : 16

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{\sqrt{x^2 + y^2}}; & (x, y) \neq (0, 0) \\ 0; & (x, y) = (0, 0) \end{cases}$$

is continuous and possesses first order partial derivatives but not differentiable at the origin.

7. Find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$ using Lagrange's method of multipliers. 16

Unit IV

8. (a) Show that the functions $u = x^2 + y^2 + z^2$, $v = xy - xz - yz$, $w = x + y - z$ are dependent. Also find the relation connecting them. 8

(b) Show that : 8

$$\boxed{m}\,\boxed{\left(m + \frac{1}{2}\right)} = \frac{\sqrt{\pi}}{2^{2m-1}}\boxed{(2m)}$$

9. (a) Find the volume of sphere $x^2 + y^2 + z^2 \leq a^2$, $a > 0$. 8

(b) Evaluate

$$\iiint x^{-1/2}.y^{-1/2}.z^{-1/2}(1 - x - y - z)^{1/2}\,dxdydz$$

extended to all positive values of the variables subject to condition $x + y + z \leq 1$. 8