(b) Expand $x^2y+3y-2$ in powers of $(x-1)$ and $(y+2)$ using Taylor's theorem upto third degree terms. **4**

Unit III

6. (a) If (a, b) is a point in the domain of definition of a function f such that :

(i) f is continuous at point (a, b)

(ii) f_y exists at point (a, b),

then f is DIFFERENTIABLE at point (a, b). **4**

(b) Give an example of a function $f(x, y)$ for which $f_{xy}(0, 0) \neq f_{yx}(0, 0)$. **4**

7. (a) Show that of all triangles inscribed in a circle, the one with the maximum area is EQUILATERAL. **4**

(b) Find the maximum and minimum values of $x^2+y^2+z^2$, subject to the conditions $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$. **4**

Roll No. .... **Exam Code : M-21**

Subject Code—52463

B. Sc. EXAMINATION

(Batch 2017 Re-appear)

(Third Semester)

MATHEMATICS

BM-231

Advance Calculus

Time : 3 Hours *Maximum Marks :* 40

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Differentiate $2^{\tan x}$ with respect to x. **1**

(b) State Euler's theorem on homogeneous functions. **1**

(c) Define implicit function with example. 1½

(d) Evaluate : 1½

$$\lim_{x \to 0} \frac{1 - \cos x}{4x^2}$$

(e) What are Serret Frenet Formulae ? 1½

(f) Find the unit normal vector to the surface : 1½

$$4x - 4y + 7z + 3 = 0$$

at the point (–1, 2, 3).

Unit I

2. (a) Using $\in-\delta$ definition, prove that $\cos^2 x$ is a continuous function. 4

(b) Verify Rolle's theorem for the function $f(x) = \left(x^2 - 4x + 3\right)e^{2x}$ in [1, 3]. 4

3. (a) State and prove Taylor's theorem with Lagrange's form of remainder. 4

(b) Evaluate : 4

$$\lim_{x \to \infty} \left(\frac{\pi}{2} - \tan^{-1} x\right)^{1/x}$$

Unit II

4. (a) Show that the function f defined by :

$$f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0 \quad , & (x, y) = (0, 0) \end{cases}$$

is continuous at (0, 0). 4

(b) If $u = x^y + x^x$, prove that : 4

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$$

5. (a) If $y^3 - 3ax^2 + x^3 = 0$, prove that : 4

$$\frac{d^2 y}{dx^2} + \frac{2a^2 x^2}{y^5} = 0$$

Unit IV

8. (a) Show that the tangent at any point of the curve whose equation referred to rectangular axes are $x = 3t,\ y = 3t^2,\ z = 2t^3$, makes a constant angle with the line $y = z - x = 0$. **4**

(b) Find the osculating plane at (x_1, y_1, z_1) on the curve of intersection of cylinders $x^2 + z^2 = a^2,\ y^2 + z^2 = b^2$. **4**

9. Find the equation of OSCULATING sphere and OSCULATING circle at (1, 2, 3) on the curve : **8**

$$x = 2t + 1,\ y = 3t^2 + 2,\ z = 4t^3 + 3$$

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