

(b) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, prove that : **8**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u (1 - 4 \sin^2 u)$$

5. (a) If $x = u^2 - v$, $y = v^2 - u$; find $\frac{\partial u}{\partial x}$ and

$$\frac{\partial u}{\partial y}. \quad \mathbf{8}$$

(b) Expand $\tan^{-1} \frac{y}{x}$ in the neighbourhood of (1, 1) by Taylor's theorem as far as terms of second degree. **8**

Unit III

6. State Schwarz's Theorem. For the function :

$$f(x, y) = \begin{cases} \frac{1}{4}(x^2 + y^2) \log(x^2 + y^2), & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Show that $f_{xy} = f_{yx}$ for all x, y but the conditions of Schwarz's theorem are not satisfied. **16**

Roll No.

Exam Code : J-21

Subject Code—52466

B. Sc. EXAMINATION

(Main/Re-appear) (Batch 2018 Onwards)

(Third Semester)

MATHEMATICS

CML-306 (Course V)

Advanced Calculus

Time : 3 Hours

Maximum Marks : 80

Note : Attempt any *Five* questions. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Test the function for continuity : **2½**

$$f(x) = \begin{cases} (x-a) \sin \frac{1}{x-a} & \text{when } x \neq a \\ 0 & \text{when } x = a \end{cases}$$

(b) If $y = \frac{u}{u-1}$ and $u = x - \frac{1}{x}$; find $\frac{dy}{dx}$. **2½**

(c) If $u = \sin^{-1} \frac{x}{y} + \tan^{-1} \frac{y}{x}$, then show that :

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0 \quad \mathbf{2\frac{1}{2}}$$

(d) Find $\frac{du}{dt}$, when $u = xy^2 + x^2y$, $x = at^2$,
 $y = 2at$. **2½**

(e) Evaluate the indeterminate form : **2**

$$\lim_{x \rightarrow 0^+} \frac{\operatorname{cosec} x}{\log x}$$

(f) Evaluate the integral in terms of Beta function : **2**

$$\int_0^1 x^m (1-x^2)^n dx, \quad m > -1, n > -1$$

(g) Evaluate the integral : **2**

$$\iint_R x dx dy, \text{ where}$$

$$R = \{(x, y) : 0 \leq x \leq a, 0 \leq y \leq x\}.$$

Unit I

2. (a) Prove that the function defined by

$$f(x) = \sin \frac{1}{x}, \quad x \in \mathbb{R}^t \text{ is continuous but}$$

not uniformly continuous on $\mathbb{R}^t$. **8**

(b) Verify Rolle's Theorem for

$$f(x) = \log(x^2 + 2) - \log 3 \text{ in } [-1, 1]. \quad \mathbf{8}$$

3. (a) Verify lagrange's mean value theorem

for $f(x) = \log x$ in $[1, e]$. **8**

(b) Prove that : **8**

$$\lim_{x \rightarrow \infty} \left(1 + \frac{k}{x}\right)^x = e^k.$$

Unit II

4. (a) Show that the function f defined by :

$$f(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

is continuous at $(0, 0)$. **8**

9. (a) Prove that the volume of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \text{ is } \frac{4}{3}nabc. \quad 8$$

- (b) Change the order of integration in

$$\int_0^a \int_y^a \frac{x \, dx \, dy}{x^2 + y^2} \text{ and hence evaluate the same.}$$

8

7. (a) Examine for maximum and minimum values the function : 8

$$xy + \frac{a^3}{x} + \frac{a^3}{y}, \quad a > 0$$

- (b) Find the minimum value of the function $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$. 8

Unit IV

8. (a) Evaluate :

$$\int_0^{n/2} \sin^n \theta \, d\theta$$

- (b) Evaluate the integral :

$$\int_0^1 \int_{\sqrt{y}}^1 dx \, dy$$

and sketch the region of integration. 8