Roll No. .... Exam Code : J-21

Subject Code—52467

B. Sc. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(Third Semester)

MATHEMATICS

CML-307

Course–VI

Numerical Analysis

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) State Newton's Forward Interpolation formula. **2**

(b) State Simpson's one-third quadrature formula. **2½**

(c) Using Newton's forward interpolation formula, write the first derivative of $f(x)$. **2½**

(d) Write Lagrange's interpolation formula. **2**

(e) Prove that :

$$\Delta \cos(cx+d) = 2\sin\frac{ch}{2}.$$

$$\cos\left(cx+d+\frac{ch+\pi}{2}\right),$$

where h is the interval of differencing. **2½**

(f) Show that equation $x^3-4x^2+7x-5=0$ has at least one positive real root and find the interval in which it lies. **2**

(g) By Newton-Raphson, derive iteration formula for finding the inverse/reciprocal of a number $N \neq 0$. **2½**

Section I

2. (a) State and prove Newton-Gregory formula for backward interpolation. **8**

(b) From the following table, find the number of students who obtained less than 45 marks : **8**

Marks	No. of Students
30–40	31
40–50	42
50–60	51
60–70	35
70–80	31

3. (a) Given the following data, find $f(x)$ as a polynomial in powers of $(x - 5)$: **8**

x	$f(x)$
0	4
2	26
3	58
4	112
7	466
9	922

(b) By means of Lagrange's formula, prove that :

$$u_1 = u_3 - 0.3(u_5 - u_{-3}) + 0.2(u_{-3} - u_{-5}).$$ **8**

Section II

4. (a) Find the value of $f(26)$ given that :

$$f(20) = 3010,$$
$$f(25) = 3979,$$
$$f(30) = 4771,$$
$$f(35) = 5441,$$
$$f(40) = 6021.$$ **8**

(b) Apply Bessel's formula to obtain y_{25}, given :

$y_{20} = 2854$, $y_{24} = 3162$, $y_{28} = 3544$, $y_{32} = 3992$. **8**

5. (a) Using Trapezoidal rule, calculate the value of the integral $\int_4^{5.2} \log_e x\,dx$ given that :

x	$\log_e x$
4.0	1.3863

4.2	1.4351
4.4	1.4816
4.6	1.5260
4.8	1.5686
5.0	1.6094
5.2	1.6486

Compare it with the exact value. **8**

(b) Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ using Simpson's $\frac{3}{8}$th rule, taking $h = \frac{1}{6}$. **8**

Section III

6. (a) Find the real root of the equation $x^3 - 5x + 3 = 0$ by secant method, correct upto three decimal places. **8**

(b) Find the real root of the equation :

$$x^4 - x - 9 = 0$$

by Newton-Raphson method, correct upto three decimal places. **8**

7. (a) Solve the following equations by triangularization method :

$$2x + y + z = 2$$

$$x + 3y + 2z = 2$$

$$3x + y + 2z = 2. \qquad \textbf{8}$$

(b) Solve the following equations by Jacobi's iteration method :

$$10x + y + 2z = 44$$

$$2x + 10y + z = 51$$

$$x + 2y + 10z = 61. \qquad \textbf{8}$$

Section IV

8. (a) Find all the eigen values of :

$$A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

and point out the smallest eigen value. **8**

(b) Using given method, reduced the martrix :

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 4 & 2 \\ 3 & 2 & 3 \end{bmatrix}$$

to tri-diagonal form. **8**

9. Use the Runge-Kutta method to solve :

$$10\frac{dy}{dx} = x^2 + y^2, \ \ y(0) = 1$$

for the interval $0 < x \leq 0.4$ with h = 0.1 **16**