Subject Code—53375

B. Sc. EXAMINATION

(Batch 2018 Onwards)

(Main & Re-appear)

(Fourth Semester)

MATHEMATICS

CML-406

Partial Differential Equations and Special Functions

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. The question paper will consist of four Sections. The students are required to attempt *one* question from each Section.

(Compulsory Question)

1. (a) Obtain a partial differential equation by eliminating the arbitrary function from : 3

$$z = f(x - ay) + g(x + ay)$$

(b) Define singular solution of a partial differential equation. 2

(c) Examine whether the equations are compatible or not : 3

$$\frac{\partial z}{\partial x} = 5x - 7y,$$

$$\frac{\partial z}{\partial y} = 6x + 8y.$$

(d) Classify the differential equation : 2

$$\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0, x \neq 0.$$

(e) Write two-dimensional Wave Equation. 2

(f) Determine the radius of convergence of power series given by $\sum_{m=0}^{\infty} \frac{1}{2^m} (x-1)^{2m}$. 2

(g) Write Legendre's Equation. 2

Section A

2. (a) Find the differential equation of all spheres whose centre lie on z-axis. **8**

(b) Solve : **8**

$$(y+z+t)\frac{\partial t}{\partial x}+(z+x+t)\frac{\partial t}{\partial y}+(x+y+t)\frac{\partial t}{\partial z}=x+y+z$$

3. (a) Solve : **8**

$$x(y^2-z^2)\,p-y(z^2+x^2)\,q=z(x^2+y^2)$$

(b) Find the complete integral of :

$$p=(qy+z)^2$$

by using Charpit's method. **8**

Section B

4. (a) Solve : **8**

$$(D^2-D'^2+D+3D'-2)\,z=e^{x-y}-x^2y$$

(b) Solve : 8

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} - x \frac{\partial z}{\partial x} = \frac{x^3}{y^2}.$$

5. (a) Solve : 8

$$(D^2 + DD' - 2D'^2)\, z = (y + 1)\, e^x$$

(b) Solve : 8

$$\frac{\partial^2 u}{\partial t^2} = C^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

subject to boundary conditions :

$$u(0, y, t) = u(a, y, t) = 0$$

for $0 < y < b,\ t > 0$

$$u(x, 0, t) = u(x, b, t) = 0$$

for $0 < x < a,\ t > 0$

and the initial conditions :

$u(x, y, 0) = f(x, y)$ and

$$\left(\frac{\partial u}{\partial t} \right)_{t=0} = 0$$

Section C

6. (a) Reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} + 2x \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

into canonical form. **8**

(b) Find the characteristic of the equation : **8**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0.$$

7. (a) Reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} + y^2 \left(\frac{\partial^2 z}{\partial y^2} \right) = y$$

into canonical form. **8**

(b) Solve : **8**

$$(q+1)\, s = (p+1)\, t.$$

Section D

8. (a) Find the power series solution of the initial value problem :

$$(x^2 - 1) \frac{d^2 y}{dx^2} + 3x \frac{dy}{dx} + xy = 0$$

where $y(2) = 4$, $y'(2) = 6$. **8**

(b) Show by the use of recurrence formula : **8**

$$4J_n''(x) = J_{n-2}(x) - 2J_n(x) + J_{n+2}(x).$$

9. (a) Find the solution of the following equation in terms of Bessel's function : **8**

$$\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + 4\left(x^2 - \frac{n^2}{x^2}\right)y = 0.$$

(b) Show that : **8**

$$P_n(-x) = (-1)^n P_n(x).$$