

Subject Code—53376

B. Sc. EXAMINATION

(Batch 2018 Onwards)

(Main & Re-appear)

(Fourth Semester)

MATHEMATICS

CML-407

Mechanics-I

Time : 3 Hours

Maximum Marks : 80

Note : Attempt *Five* questions in all. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Three coplanar forces acting on a particle are in equilibrium. The angle between the first and the second is 60° and that

between the second and third is 150° .
Find the ratio of the magnitude of the forces. $2\frac{1}{2}$

- (b) A man carries a bundle at the end of a stick which is placed horizontally over his shoulder. If the distance between his hand and his shoulder be changed, prove that the pressure on his shoulder varies inversely as the distance between his hand and shoulder. $2\frac{1}{2}$

- (c) The position of a moving point at time t is given by $x = a \cos t$ and $y = a \sin t$.
Find its path and velocity. $2\frac{1}{2}$

- (d) Define axis of a couple. $2\frac{1}{2}$

- (e) Define null lines and null plane. $2\frac{1}{2}$

- (f) A body of mass 150 kg. is acted upon by a force of 5 N for 15 minutes. If the particle starts from rest, find its velocity at the end of this period. $2\frac{1}{2}$

- (g) What are the Kepler's laws of planetary motion ? 2

Unit I

2. (a) Two forces $P + Q$ and $P - Q$ make an angle 2α with one another and their resultant makes an angle θ with the bisector of the angle between them. Show that $P \tan \theta = Q \tan \alpha$. 8
- (b) Show that if three parallel forces acting on a rigid body are in equilibrium, then each is proportional to the distance between the other two. 8
3. (a) Three forces, P, Q, R act along the sides BC, CA and AB of a triangle ABC . Show that if their resultant passes through :
- (i) the incentre, then $P + Q + R = 0$
- (ii) the centroid, then $P \operatorname{cosec} A + Q \operatorname{cosec} B + R \operatorname{cosec} C = 0$. 8
- (b) $ABCDEF$ is a regular hexagon. Forces, $P, 2P, 3P, 2P, 5P, 6P$ act along AB, BC, DC, ED, EF, AF respectively. Show that six forces are equivalent to a couple and find its moment. 8

Unit II

4. (a) A force P acts along the axis of x and another force $3P$ along a generator of the cylinder $x^2 + y^2 = a^2$. Show that the central axis lies on the cylinder $9(3x - z)^2 + 100y^2 = 81a^2$. 8
- (b) Show that every given system of forces acting on a rigid body can be reduced to a wrench. 8
5. (a) Show that any wrench may be resolved into two wrenches, whose axes intersect at right angles, in an infinite number of ways. 8
- (b) Find the null point of the plane $lx + my + nz = 1$ for the system of forces $(X, Y, Z; L, M, N)$. 8

Unit III

6. (a) A particle moves along a circle $r = 2a \cos \theta$ in such a way that its acceleration towards origin is always zero. Show that the transverse acceleration varies as the fifth power of $\operatorname{cosec} \theta$. 8

- (b) Prove that if the tangential and normal accelerations of a particle describing a plane curve be constant throughout the motion, the angle ψ through which the direction of motion turns is given by $\psi = A \log (1 + Bt)$.
7. (a) A ship steams due west with a velocity of 15 km/hr relative to the current which is flowing at the rate of 6 km/hr due south. What is the velocity of a train going north at a rate of 30 km/hr relative to the ship ? 8
- (b) If the displacement of a moving point at any time t is given by an equation of the form $x = a \cos kt + b \sin kt$, show that the point executes S.H.M. If $a = 3$, $b = 4$ and $k = 2$; find the period, amplitude and maximum velocity of the motion. 8

Unit IV

8. (a) Two particles of masses m_1 and m_2 are connected by a light inextensible string; m_2 is placed on the smooth horizontal

table. The string passes over a light pulley at the edge of the table and m_2 is hanging freely. Find the motion, the tension in the string and the pressure on the pulley. 8

- (b) A particle describes the equiangular spiral $r = ae^{\theta \cot \alpha}$ under a force to the pole. Find the law of force. 8

9. (a) A particle moves in a plane under a central force which varies inversely as the square of the distance from the fixed point. Find the orbit. 8

- (b) If a planet were suddenly stopped in its orbit when at a distance ' a ' from the sun, show that it would fall in the sun in time

$$\frac{\sqrt{2}\pi a^{3/2}}{4\sqrt{\mu}} \text{ which is } \frac{\sqrt{2}}{8} \text{ times the period}$$

for the planet's revolution. 8