

Roll No. 223112320027  
Hari

Exam Code : M-24

Subject Code—61374

**B. Sc. EXAMINATION**

(Batch 2022 Onwards)

(Fourth Semester)

MATHEMATICS

CML-406 (Main)

Partial Differential Equations and Special  
Functions

*Time : 3 Hours*

*Maximum Marks : 80*

**Note :** Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory. All questions carry equal marks.

**(Compulsory Question)**

1. (a) Define degree of a partial differential equation. Given an example of a partial differential equation of degree 4. 2

(1-09-47-0524) J-61374

P.T.O.

<https://t.me/BscNonmedical2023>

(b) Solve the partial differential equation

$p^2 - q^2 = k^2$ , where  $k$  is a constant,

$$p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}. \quad 2$$

(c) Find particular integral of : 2

$$x^2y - 3xys + 2y^2t + px + 2qy = x + 2y$$

$$\text{where } p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}, r = \frac{\partial^2 z}{\partial x^2},$$

$$s = \frac{\partial^2 z}{\partial x \partial y} \text{ and } t = \frac{\partial^2 z}{\partial y^2}.$$

(d) Define linear homogeneous partial differential equation of order 'n'. 2

(e) Define Cauchy's problem for the second order partial differential equation. 2

(f) Find the characteristic curves of the following differential equation : 2

$$(1+x^2) \frac{\partial^2 z}{\partial x^2} + (1+y^2) \frac{\partial^2 z}{\partial y^2} + x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 0.$$

J-61374

2

- (g) Write generating function for  $P_n(x)$ , where  $P_n(x)$  is Legendre polynomial of order 'n'. 2

- (h) Prove that :

$$\int J_3(x) dx = c - J_2(x) - \frac{2}{x} J_1(x)$$

where  $J_n(x)$  is the Bessel function of order 'n'. 2

### Section I

2. (a) Form a partial differential equation by eliminating arbitrary constants from  $z = ax + by + cxy$ . 8

- (b) Solve :

$$(x^2 + y^2 + yz) p + (x^2 + y^2 - zx) q = z(x + y)$$

where  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$ . 8

3. (a) Solve the differential equation :

$$zp - zq = x + y$$

where  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$ . 8

- (b) Find the complete integral of :

$$p^2 - qy^2 = y^2 - x^2$$

by Charpit's method, where  $p = \frac{\partial z}{\partial x}$  and

$$q = \frac{\partial z}{\partial y}. \quad 8$$

### Section II

4. (a) Solve : 8

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \cos 2x \cos 3y.$$

- (b) Solve the differential equation : 8

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy.$$

J-61374

5. (a) Solve :

$$(D^3 - 7DD'^2 - 6D'^3) z =$$

$$x^2 + xy^2 + y^3 + \cos(x - y)$$

where  $D = \frac{\partial}{\partial x}$  and  $D' = \frac{\partial}{\partial y}$ . 8

(b) Find the general solution of : 8

$$x^2 \frac{\partial^2 z}{\partial x^2} - 4y^2 \frac{\partial^2 z}{\partial y^2} - 4y \frac{\partial z}{\partial y} - z = x^2 y^2.$$

### Section III

6. (a) Reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} - x^2 \frac{\partial^2 z}{\partial y^2} = 0$$

to the canonical form. 8

(b) Solve the equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = 0$$

subject to the initial conditions

$$z(x, 0) = f(x) \text{ and } \left[ \frac{\partial z}{\partial y} \right]_{y=0} = g(x). \quad 8$$

7. Classify and reduce the equation :

$$y^2 \frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial x^2} = 0$$

to the canonical form and hence solve it, where  
 $x \neq 0$  and  $y \neq 0$ . 16

#### Section IV

8. (a) Find the power series solution of :

$$\frac{d^2 y}{dx^2} + (x-3) \frac{dy}{dx} + y = 0$$

about  $x = 2$ . 8

- (b) State and prove Rodrigue's formula for Legendre polynomial. 8

9. (a) Prove that :

$$e^{\frac{x}{2}\left(t - \frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x) t^n$$

where  $J_n(x)$  is the Bessel function of order ' $n$ '. 8

(b) Express :

$$4x^3 - 2x^2 - 3x + 8$$

in terms of Legendre's polynomial. 8

<https://t.me/BscNonmedical2023>

(1-09-50-0524) J-61374

7

1250