Roll No. .... Exam Code : M-23

Subject Code—59290

B. Sc. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(Fourth Semester)

MATHEMATICS

CML-406

Partial Differential Equations and Special Functions

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

1. (a) Define a partial differential equation and its order and degree. **3**

(b) Solve $\frac{\partial^2 u}{\partial x \partial t} = e^{-t} \cos x$.

(c) Write down the steps for reducing a parabolic equation to its canonical form. 2

(d) Write down one dimensional heat and wave equations. 2

(e) Find the real characteristic of the equation

$$\frac{\partial^2 z}{\partial x^2}+4\frac{\partial^2 z}{\partial x \partial y}+4\frac{\partial^2 z}{\partial y^2}=0.$$ 3

(f) Show that equation $\frac{\partial z}{\partial x}=5x-7y$ and $\frac{\partial z}{\partial y}=6x+8y$ are not compatible. 2

(g) Show that :

$$J_{-\frac{3}{2}}(x)=-\sqrt{\frac{2}{\pi x}}\left(\frac{\cos x}{x}+\sin x\right).$$ 2

Section I

2. (a) Find the differential equations of all planes which are at a constant distance 'a' from the origin. 8

(b) Solve the partial differential equation :

$$p\cos(x+y)+q\sin(x+y)=z+\frac{1}{z}. \quad 8$$

3\. (a) Find the complete integral of :

$$p=(qy+z)^2$$

by using Charpit's method. 8

(b) Find the complete integral of :

$$(x_2+x_3)(p_2+p_3)^2+zp_1=0$$

by using Jacobi's method. 8

Section II

4\. (a) Solve the differential equation :

$$\frac{\partial^3 z}{\partial x^3}-3\frac{\partial^3 z}{\partial x^2\partial y}+4\frac{\partial^3 z}{\partial y^3}=e^{x+2y}. \quad 8$$

(b) Solve :

$$\frac{1}{x^2}\frac{\partial^2 z}{\partial x^2}-\frac{1}{x^3}\frac{\partial z}{\partial x}=\frac{1}{y^2}\frac{\partial^2 z}{\partial y^2}-\frac{1}{y^3}\frac{\partial z}{\partial y}. \quad 8$$

5. Find the solution of two dimensional Laplace equation $\nabla^2 u = 0$ in Cartesian coordinates in the region $0 \le x \le a$, $0 \le y \le b$ satisfying the conditions :

$$u(0, y) = u(a, y) = u(x, b) = 0$$

and $$u(x, 0) = x(a - x), 0 < x < a.$$ 16

Section III

6. (a) Classify and reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} + y^2\left(\frac{\partial^2 z}{\partial y^2}\right) = y$$

to canonical form. 8

(b) Solve :

$$ry^2 + 2xys + x^2 t + px + qy = 0.$$ 8

7. (a) Solve :

$$2qr + 2qt - 4pq(rt - s^2) = 1.$$ 8

(b) Show that the condition of the Cauchy problem for the equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{1}{c^2}\left(\frac{\partial^2 z}{\partial t^2}\right) = 0,\ c > 0$$

satisfying $z(x, 0) = f(x)$

and $\left[\frac{\partial z}{\partial t}\right]_{t=0} = 0$ is :

$$z(x, t) = \frac{1}{2}\{f(x - ct) + f(x + ct)\}.$$ 8

Section IV

8. (a) Find the power series solution of the following differential equation in powers of x (i.e. about $x = 0$)

$$(x^3 - 1)\frac{d^2 y}{dx^2} + x^2 \frac{dy}{dx} + xy = 0.$$ 8

(b) Find the solution of the following equation in terms of Bessel's function :

$$\frac{d^2 y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + 4\left(x^2 - \frac{n^2}{x^2}\right)y = 0$$ 8

9. (a) Verify that the Bessel's function $J_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}}\sin x$ satisfies the Bessel's equation of order $\frac{1}{2}$. 8

(b) Prove that :

$$\int_{-1}^{1} x^2 P_{n+1}(x) P_{n-1}(x)\, dx = \frac{2n(n+1)}{(2n-1)(2n+1)(2n+3)}.$$

Hence deduce the value of :

$$\int_{0}^{1} x^2 P_{n+1}(x) P_{n-1}(x)\, dx.$$ 8