Section III

6. (a) Obtain the Fourier expansion of the function defined by :

$$f(x)=\begin{cases}1+\dfrac{2x}{\pi} & \text{when } -\pi\le x\le 0\\[2mm] 1-\dfrac{2x}{\pi} & \text{when } 0\le x\le \pi\end{cases}$$

(b) Obtain the Fourier series for the function :

$$f(x)=\begin{cases}\pi x, & 0\le x\le 1\\ \pi(2-x), & 1\le x\le 2\end{cases}$$

7. (a) Obtain a sine series for the function :

$$f(x)=\begin{cases}x, & \text{for } 0<x<\dfrac{\pi}{2}\\[2mm] 0, & \text{for } \dfrac{\pi}{2}<x<\pi\end{cases}$$

(b) Given the series $2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx$, show that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ using the Parseval's Identity.

Subject Code—52627

B. Sc. EXAMINATION

(Batch 2018) (Main)

(Fifth Semester)

MATHEMATICS

CML-507 (i) (Paper II)

Sequence & Series

Time : 3 Hours *Maximum Marks :* 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Define closed sets.

(b) Define Null sequence.

(c) Show that the series $\sum_{n=1}^{\infty} \frac{n}{2n+1}$ is not convergent.

(d) State Gauss Test.

(e) Define Half range series.

(f) Define Refinement.

(g) If $a_n \le b_n$ for all n and $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, then prove that :

$$a \le b$$

Section I

2. (a) If x and y are two positive real numbers, then there exists a natural number n such that $nx > y$.

(b) Prove that the union of an arbitrary family of open sets is an open set.

3. (a) Show that the sequence $< a_n >$ where $a_n = x^n$ converges to 0 if $|x| < 1$.

(b) Let $< a_n >$ be a sequence such that $a_n \ne 0$ for all $n \in \mathrm{N}$ and $\frac{a_{n+1}}{a_n} \to l$. If $|l| < 1$, then $\lim_{n\to\infty} a_n = 0$.

Section II

4. (a) Test the convergence of the series :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots$$

(b) State and prove Raabe's test.

5. (a) Using integral test, test the behaviour of the series :

$$\sum_{n=1}^{\infty} \frac{1}{n^p},\ p > 0$$

(b) Show that :

$$\sum_{1}^{\infty} \frac{(-1)^{n-1}}{n}\left(1+\frac{1}{n}\right)^{-n}$$

Section IV

8. (a) If a function is continues on [a, b], then it is integrable on [a, b].

(b) If f is bounded and integrable on [a, b], $|f|$ is also bounded and integrable on [a, b]. Moreover, $\left|\int_a^b f\,dx\right| \le \int_a^b |f|\,dx$.

9. (a) State and prove mean value theorem of integral calculus.

(b) Prove that :

$$\frac{\pi^2}{9} \le \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{\sin x}\,dx \le \frac{2\pi^2}{9}$$

Section IV

8. (a) If a function is continues on [a, b], then it is integrable on [a, b].

(b) If f is bounded and integrable on [a, b], $|f|$ is also bounded and integrable on [a, b]. Moreover, $\left|\int_a^b f\,dx\right| \le \int_a^b |f|\,dx$.

9. (a) State and prove mean value theorem of integral calculus.

(b) Prove that :

$$\frac{\pi^2}{9} \le \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{\sin x}\,dx \le \frac{2\pi^2}{9}$$