7. (a) Prove that a field has no proper ideals. 8

(b) Let R be a ring of 2×2 matrices over integers. Let

$$S = \left\{ \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} : a, b \text{ are integers} \right\}.$$

Then S is a left ideal but not right ideal.

Unit IV

8. (a) Define unit element, prime element and irreducible element. Also prove that if 'a' is a unit of a commutative ring R with unity element, then a^{-1} is also a unit in R. 8

(b) Find all the units of $Z = \left[\sqrt{-5}\right]$. 8

9. (a) If F is a field then prove that F[x] may not be a field. 8

(b) Show that the polynomial $x^3 + y^3 + xy$ is irreducible over C, the field of complex numbers. 8

Roll No. .... Exam Code : M-21

Subject Code—52625

B. Sc. EXAMINATION

(Batch 2018) (Main)

(Fifth Semester)

MATHEMATICS

CML-506(i) Paper-I

Groups and Rings

Time : 3 Hours *Maximum Marks : 80*

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Questions

1. (a) Find the order of each element of the multiplicative group $\{1, -1, i, -i\}$. 2½

(b) Define cyclic group. 2

(c) Give an example of a ring with zero divisors. 2½

(d) Find the inverse of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$ in group S_6. **2**

(e) Define Euclidean ring and give one example. **2½**

(f) Define Prime ideal and Maximal ideal. **2**

(g) Define inner automorphism with an example. **2½**

Unit I

2. (a) A non-empty subset H of a group G is a subgroup of G iff $a, b \in H \Rightarrow ab^{-1} \in H$. **8**

(b) Every subgroup of a cyclic group is cyclic. **8**

3. (a) Let G be the additive group of integers and H be the subgroup of G obtained on multiplying each element of G by 3. Find cosets of H. **8**

(b) Write all elements of symmetric group S_3 as product of disjoint cycles. **8**

Unit II

4. (a) State and prove Lagrange's theorem. **8**

(b) A subgroup H of a group G is normal if and only if each left coset of H in G is a right coset of H in G. **8**

5. (a) Prove the first theorem of isomorphism. (Every homomorphic image of a group G is isomorphic to some quotient group of G). **8**

(b) Let G be a non-abelian group such that $o(G) = p^3$, where p is prime. Show that $o(Z(G)) = p$. **8**

Unit III

6. (a) Prove that a division ring (skew field) has no zero divisors. **8**

(b) Prove that centre of a ring R is a subring of R. **8**