Roll No. .... Exam Code : J-22

Subject Code—55638

B.Sc. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(Fifth Semester)

MATHEMATICS

CML-506(i) Paper I

Groups and Rings

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. **1** is compulsory.

(Compulsory Question)

1. (a) Show that the set $S = \{1, 2, 3, 4, 5\}$ is not a group under the composition 'multiplication modulo 6'. **2**

(b) State Lagrange's theorem. 2

(c) Write the permutation :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 3 & 1 & 4 & 8 & 6 & 9 & 7 & 5 \end{pmatrix}$$

as the product of disjoint cycles. 2

(d) With the help of an example show that the inner automorphism corresponding to two distinct elements of a group may be same. 2

(e) Prove that the ring {0, 1, 2, 3, 4, 5} with addition modulo 6 and multiplication modulo 6 as compositions is a ring with zero divisors. 2

(f) Define Principal Ideal Domain. 2

(g) If a is a unit of a commutative ring R with unity element, then a^{-1} is also a unit in R. 2

(h) If $-2i$ is a root of $f(x) = x^4 + 3x^3 + 6x^2 + 12x + 8$, then solve the equation completely. 2

Section I

2. (a) Show that the union of two subgroups is a subgroup if and only if one is contained in the other. 8

   (b) Show that every finite group of prime order is cycle. 8

3. (a) Show that order of every element of a finite group is a divisor of the order of the group. 8

   (b) A subgroup H of a group G is normal if and only if each left coset of H in G is a right coset of H in G. 8

Section II

4. (a) Show that every homomorphic image of a group G is isomorphic to some quotient group of G. 8

   (b) The set Inn(G) of all iner automorphisms of a group G is isomorphic to the quotient group G|Z(G), where Z(G) is centre of G. 8

5. (a) If G' is a commutator subgroup of G, then : 8

   (i) G' is a normal subgroup of G

   (ii) $G|G'$ is abelian

   (iii) G' is the smallest subgroup of G such that $G|G'$ is abelian.

   (b) Let $S = \{1, 2, 3, 4, 5\}$. If $f = (2, 3)$, $g(1, 4)$, show that : 8

$$fog = gof$$

Section III

6. (a) Show that a division ring has no zero divisors. 8

   (b) Show that the ring of integers is a principal ideal ring. 8

7. (a) Let R be a commulative ring and S is an ideal of R. Then $R|S$ is an integral domain iff S is a prime ideal. 8

   (b) Let D_1 and D_2 be two isomorphic integral domains. If F_1 and F_2 are their respective fields of quotients, then show that F_1 and F_2 are also isomorphic. 8

Section IV

8. (a) An element in a principal ideal domain is prime element iff it is irreducible. **8**

(b) Let R be a Euclidean ring and a, b be two non-zerio elements of R, then :

(i) if b is a unit in R, then $d(ab) = d(a)$

(ii) if b is not a unit in R, then $d(ab) > d(a)$. **8**

9. (a) Show that if 'a' is an irreducible element of a unique factorization domain R, then 'a' must be prime. **8**

(b) Let R be an integral domain with unity. Show that if a is an irreducible element of R, then a is irreducible element R[x]. **8**