Roll No. 22311232 0007
Hari

Exam Code : D-24

Subject Code—62454

B.Sc. EXAMINATION

(Main/Re-appear)

(Batch 2022 Onwards)

(Fifth Semester)

MATHEMATICS

CML-506(i) Paper I

Groups and Rings

Time : 3 Hours *Maximum Marks :* 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) If every element of a group is its own inverse then show that the group is abelian. 2

(b) If (G, $\cdot$) is a group and a, b are any element of G, then order of ab and ba are same. 2

(c) Every subgroup of an abelian group is always normal. 2

(d) Let G be a cyclic group of order 4, show that the group of automorphisms of G is of order 2. 2

(e) Prove that the set of integers is a subring of ring of rational numbers. 2

(f) Prove that a division ring is a simple ring. 2

(g) Prove that any homomorphism of a field is either one-one or takes each other. 2

(h) Show than an element a in an Euclidean ring R is a unit iff $d(a) = d(1)$. 2

Unit I

2. (a) If H and K are two subgroups of a group G, then HK is a subgroup of G iff HK = KH. 8

(b) Order of a cyclic group is the order of its generator. 8

3. (a) The order of the element a and $x^{-1}ax$ are same where a, x are the two elements of a group. 8

(b) Show that if G is an infinite cyclic group, then G has exactly two generators. 8

Unit II

4. (a) The subgroup H of a group G is normal if and only if each left coset of H in G is a right coset of H in G. 8

(b) If f is an automorphism of a group G and $a \in G$ be an element, then show that $f(N(a)) = N(f(a))$. 8

5. (a) If P is a prime number and G is a non abelian group of order P^3, show that Z(G) has exactly P elements. 8

(b) Every homomorphic image of a group G is isomorphic to some quotient group of G. 8

Unit III

6. (a) Prove that the set {0, 1, 2, 3, 4, 5} with addition modulo 6 and multiplication modulo 6 as composition is a ring. **8**

   (b) An arbitrary intersection of ideals of a ring is an ideal of the ring. **8**

7. (a) Every finite non zero integral domain is a field. **8**

   (b) Let S, T be two ideals of a ring R. Then $(S+T)/S \cong T/(S \cap T)$. **8**

Unit IV

8. (a) The integral domain $<z, +, \cdot>$ of integers is an Euclidean domain. **8**

   (b) Show that $\sqrt{-5}$ is a prime element of the ring $z\sqrt{-5} = \left\{a + \sqrt{-5}\, b : a, b \in z\right\}$. **8**

9. (a) If R is an integral domain, then R[x] is also an integral domain. **8**

   (b) show that the polynomial $x^4 + 1$ is irreducible over Q. **8**