Roll No. .... Exam Code : D-22

Subject Code—58473

B. Sc. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(Fifth Semester)

MATHEMATICS

CML–506(i) Paper–I

Groups and Rings

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Define centre of a group.

(b) What is the inverse of cycle (1 4 2 6) ?

(c) Show that the ring :

$$Z_5 = \{(0, 1, 2, 3, 4,), +_5, \times_5\}$$

has no proper ideal.

(d) Show that the polynomial $x^2 - 6x + 2$ is irreducible over Q.

(e) Give an example of a ring with zero divisors.

(f) Prove that the set of all odd integers is not a ring.

(g) Define Euclidean ring.

Unit I

2. (a) If $a \in G$ is a element such that $a^m = e$ and order$(a) = n$, then $n \mid m$ (where G is a group).

(b) Prove that a group of order 37 is cyclic.

3. (a) If H & K are finite subgroups of a group G, then show that :

$$O(HK) = \frac{O(H)\,O(K)}{O(H \cap K)}.$$

(b) Show that the set of the inverses of the elements of a right coset is a left coset.

Unit II

4. (a) Show by an example that the inner automorphism corresponding to two different elements of a group may be same.

(b) Find the automorphism group of integers w.r.t. binary operation of addition.

5. (a) Prove that if G is a group then $G/Z(G) \cong \text{Inn}(G)$.

(b) Show that $x \longrightarrow x^{-1}$ is an outo-morphism iff G is abelion.

Unit III

6. (a) Prove that the ring of integers is a principal ideal ring.

(b) Prove that a field is a division ring and simple ring.

7. (a) If R is an integral domain and $a, b \in I$ such that $a^m = b^m$ and $a^n = b^n$, where m, n are co-prime then prove $a = b$.

(b) Show that a commutative ring without zero division can be embedded into a field.

Unit IV

8. (a) Show that an element of a P.I.D. is prime element if it is irreducible.

(b) Prove that if R is an integral domain then R $[x]$ is also an integral domain.

9. (a) Prove that every Euclidean ring is a U.F.D.

(b) State and prove Gauss lemma.