Roll No. .... Exam Code : D-22

Subject Code—58475

B.Sc. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(Fifth Semester)

MATHEMATICS

CML-507(i), Paper-II

Sequence and Series

Time : 3 Hours *Maximum Marks : 80*

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Prove that the smallest member of a set, if it exists, must be the infimum of the set.

(b) Define Divergent Sequence.

(c) Define p-series Test. 2

(d) State Logarithmic Test

(e) Define Dirichlet's Test.

(f) State Dirichlet's Conditions for a Fourier Expansion.

(g) Define Refinement of a Partition.

(h) State First Mean Value Theorem. 2

Unit I

2\. (a) State and prove Bolzano-Weierstrass Theorem. 8

(b) The union of finite number of closed sets is a closed set. 8

3\. (a) Let $< a_n >$ be a sequence such that $a_n \neq 0$ for all $n \in \mathrm{N}$ and $\frac{a_{n+1}}{a_n} \to l$. If $|l| < 1$, then $\lim_{n\to\infty} a_n = 0$.

(b) Prove that the sequence $< a_n >$ defined by $a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots + \frac{1}{n!}$ is convergent and $2 \leq \lim_{n \to \infty} a_n \leq 3$.

Unit II

4. (a) A necessary and sufficient condition for a series $\sum_{n=1}^{\infty} a_n$ to converge is that to each $\varepsilon < 0$, there exists a positive integer m such that $|a_{m+1} + a_{m+2} + \ldots.. + a_n| < \varepsilon$ for all $n \geq m$.

(b) Test the convergence of the series :

$$\frac{1^2}{2^2} + \frac{1^2.3^2}{2^2.4^2} x + \frac{1^2.3^2.5^2}{2^2.4^2.6^2} x^2 + \ldots.. \ (x > 0).$$

5. (a) Using Cauchy's integral test, discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$, $p > 0$. 8

(b) Show the the series

$$\frac{\log 2}{2^2} - \frac{\log 3}{3^2} + \frac{\log 4}{4^2} - \ldots \text{ converges.}$$

Unit III

6. (a) If the series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

converges uniformly to a function 'f' on $[-\pi, \pi]$, then it is the Fourier series for 'f' on $[-\pi, \pi]$.

(b) Find the Fourier expansion for the function $f(x) = e^x$ in $-\pi < x < \pi$. Hence deduce that :

$$\frac{\pi}{\sinh \pi} = 1 + 2\sum_{n=1}^{\infty} \frac{(-1)^n}{1+n^2}.$$

7. (a) Obtain the Fourier series expansion of the function f in $(0, 2\pi)$ defined as :

$$f(x) = \begin{cases} x, & \text{for } 0 < x < \pi \\ 0, & \text{for } \pi < x < 2\pi \end{cases}$$

Also find the sum of series at $x = \pi$.

(b) Obtain a cosine series for the function $f(x) = \pi - x$ in $0 < x < \pi$.

Unit IV

8. (a) If f is a bounded function on $[a, b]$ and P' is a refinement of a partition P of $[a, b]$, then :

   (i) $L(f, P') \geq L(f, P)$

   (ii) $U(f, P') \leq U(f, P)$.

   (b) If a function f is defined on $[0, a]$, $a > 0$ by $f(x) = x^3$, then show that f is Riemann integrable on $[0, a]$ and $\int_0^a f dx = \frac{a^4}{4}$.

9. (a) By definition, prove that

$$\int_0^a \cos x dx = \sin a,$$

   where a is a fixed number.

   (b) If f and g are bounded and integrable functions on $[a, b]$, with $f(x) \leq g(x)$, $x \in [a, b]$, then $\int_a^b f dx \leq \int_a^b g dx$.