

Subject Code—58477

B. Sc. EXAMINATION

(Main/Re-appear) (Batch 2018 Onwards)

(Fifth Semester)

MATHEMATICS

CML-508(i) Paper III

Number Theory & Trigonometry

Time : 3 Hour's

Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 9 is compulsory.

Section I

1. (a) State and prove Linear Diophantine equation. 8

(b) Solve the congruence : 8

$$222x \equiv 12 \pmod{18}$$

2. (a) State and prove Chinese remainder theorem. 8
- (b) Show that $n^{16} - a^{16}$ is divisible by 85 if n and a are co-prime to 85. 8

Section II

3. (a) For $n > 2$, prove $\phi(n)$ is an even integer. 8
- (b) Show that :

$$\phi(n) = \phi(n+1) = \phi(n+2)$$
for $n = 5186$. 8
4. (a) Let a be any integer and m be a positive integer. If $(a, m) = 1$, then

$$a^{\phi(m)} \equiv 1 \pmod{m}. \quad 8$$
- (b) To show that Mobius function is multiplicative function. 8

Section III

5. (a) Explain in detail about composite numbers having primitive roots. 8

(b) Solve the congruence : 8

$$x^2 \equiv 5 \pmod{29}$$

6. (a) Prove the property of Legendre symbol :

$$\left(\frac{ab}{p}\right) = \left(\frac{a}{p}\right)\left(\frac{b}{p}\right) \quad 8$$

(b) State and prove quadratic reciprocity law.

8

Section IV

7. (a) Prove : 6

$$16 \sin^5 \theta = \sin 5\theta - 5 \sin 3\theta + 10 \sin \theta$$

(b) If $\tan(\theta + i\phi) = \sin(x + iy)$, prove that :

$$\coth y \sinh 2\phi = \cot x \cdot \sin 2\theta \quad 10$$

8. (a) Solve the equation : 6

$$\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$$

(b) Find the sum of sines of n angles when the angles are in Arithmetical Progression (A.P.) 10

Compulsory Question

9. (a) Find real and imaginary part of i^i .
- (b) Write both the forms of Gregory series.
- (c) Define Euler's criterion.
- (d) Explain primitive roots for Primes.
- (e) Explain totally multiplicative functions.
- (f) Define greatest integer function with example.
- (g) State Wilson's theorem.
- (h) Explain prime counting function. $2 \times 8 = 16$