Roll No. 19311230013

Exam Code : J-22

Subject Code—55640

B. Sc. EXAMINATION

(Main/Reappear) (Batch 2018 Onwards)

(Fifth Semester)

MATHEMATICS

CML-507 (i)

Paper–II

Sequences and Series

Time : 3 Hours *Maximum Marks : 80*

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Define interior point of a set and give example of a set which has no interior point. 3

(b) Prove that if the series $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$. 3

(c) State Cauchy's root test. 2

(d) Test the absolute convergence of infinite series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^2}$. 2

(e) Find the Fourier coefficient a_n for the function $f(x)=x$ in $[-\pi, \pi]$. 2

(f) Obtain the cosine series for the function $f(x)=\pi-x$ in $0<x<\pi$. 2

(g) Examine the convergence of $\int_1^{\infty}\frac{dx}{x}$. 2

Section I

2\. (a) Prove that the union of an arbitrary family of open sets is an open set. 8

(b) The arbitrary union of closed sets may not be a closed set. Prove. 8

3. (a) Show that :

$$\underset{n\to\infty}{\text{limit}}\left[\frac{1}{n^2}-\frac{1}{(n+1)^2}+\ldots+\frac{1}{(2n)^2}\right]=0. \qquad 8$$

(b) Prove that $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3. **8**

Section II

4. (a) Show that the series $\sum_{n=1}^{\infty}\left(\frac{1}{n^2}\right)^{1/n}$ is divergent. **8**

(b) Discuss the convergence of the series :

$$\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^n \cdot x^n,\ x>0. \qquad 8$$

5. (a) Test the convergence of the series :

$$x+\frac{2^2\cdot x^2}{2!}+\frac{3^3\cdot x^3}{3!}+\frac{4^4\cdot x^4}{4!}+\ldots. \qquad 8$$

(b) Discuss the convergence and absolute convergence of the series :

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}\left(\sqrt{n+1}-\sqrt{n-1}\right). \qquad 8$$

Section III

6. (a) Find the Fourier series expansion of the function $f(x)=x-x^2$ in $[-\pi, \pi]$.

Hence show that :

$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots=\frac{\pi^2}{12}. \qquad 8$$

(b) Obtain the Fourier series expansion of the function f in $(0, 2\pi)$ defined by :

$$f(x)=\begin{cases} x \text{ for } & 0<x<\pi \\ 0 \text{ for } & \pi<x<2\pi \end{cases}.$$

Also find the sum of the series at $x=\pi$.

8

7. (a) Express $f(x)=x$ as a half range cosine series in $0<x<2$ and also as a half range sine series too in $0<x<2$. 8

(b) Given the series $x = 2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin nx$,

show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, using Parserval's identity. 8

Section IV

8. (a) Show that the function f defined on $[0, 1]$ by :

$$f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ -1 & \text{if } x \text{ is irrational} \end{cases}$$

is bounded but not R-integrable. 8

(b) Prove that every bounded monotonic function is an integrable function. 8

9. (a) Prove that :

$$\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$$

using Mean Value theorem. 8

(b) Prove that if a function f is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\,dt$, then f is differentiable on $[a, b]$ and $F' = f$. 8