(b) Prove that every finite ring has non-zero characteristic. **4**

7. (a) Prove that if every ideal of a commutative ring with unity is a prime ideal, then R is a field. **4**

(b) Prove that an integral domain can be embedded into a field. **4**

Section IV

8. (a) Prove that every ideal of Euclidean ring is a principle ideal. **4**

(b) Find all the units of $Z\left[\sqrt{-7}\right]$ and $Z\left[\sqrt{-1}\right]$. **4**

9. (a) Prove that in a UFD (Unique factorization domain) an element is prime iff it is irreducible. **4**

(b) State and prove Gauss Lemma. **4**

Subject Code—52471

B. Sc. EXAMINATION

(Batch 2017) (Reappear)

(Fifth Semester)

MATHEMATICS

BM-352

Groups and Rings

Time : 3 Hours *Maximum Marks :* 40

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) State third theorem of isomorphism for groups. **2**

(b) State Eienstein's criterion of irreducibility. **2**

(c) Give an example of a division ring which is not a field. Justify your answer. **2**

(d) If I is an ideal of R such that $1 \in I$ where 1 is unit of R, then I = R. **2**

Section I

2. (a) If the element 'a' of a group G is of order n, then $a^m = e$ iff n/m. **4**

(b) The number of generators of a finite cyclic group of order n is $\phi(n)$, where $\phi(n)$ is the Euler's ϕ function. **4**

3. (a) A subgroup H of a group G is normal if and only if the product of two right cosets of H is again a right coset of H is G. **4**

(b) Prove that order of every element in a finite group divides order of group. **4**

Section II

4. (a) If H and K are two normal subgroups of a group G, such that $H \subseteq K$, then $\frac{G}{K} \cong \frac{G/H}{K/H}$. **4**

(b) Let $f : G \to G$ be a homomorphism. Let f commutes with every inner automorphism of G. Show that the set $H = \{x \in G \mid f^2(x) = f(x)\}$ is a normal subgroup of G. **4**

5. (a) Define derived group $\delta(G)$ of a group G and prove that $\frac{G}{\delta(G)}$ is abelian. **4**

(b) Find the derived group $\delta(G)$ of group 'G', where G is the group of permutations on a set 'S' of 3 elements. **4**

Section III

6. (a) Prove that $Z_{10} = \{0, 1, 4 .... 9\}$ does not form a field w.r.t. multiplication and addition moduls 10, whereas $Z_p = \{0, 1, 4 .... p-1\}$ form a field w.r.t. multiplication and addition moduls p, if p is a prime number. **4**