Roll No. .... Exam Code : M-21

Subject Code—52629

B. Sc. EXAMINATION

(Batch 2018) (Main)

(Fifth Semester)

MATHEMATICS

CML-508(i) Paper-III

Number Theory and Trigonometry

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. All questions carry equal marks.

Compulsory Question

1. (a) Prove that : **3**

$$\cosh^2 x - \sinh^2 x = 1 \cdot$$

(b) Find $d(n)$ and $\sigma(n)$ for $n = 250$, where $d(n)$ denotes the number of positive divisors of n and $\sigma(n)$ denotes the sum of the positive divisors of n. **2**

(c) Define the primitive roots of a number. **2**

(d) Find all possible values of n which satisfies $\phi(n) = 23$. **2**

(e) Define Legendre symbol (a/p); where p be an odd prime and $gcd\ (a, p) = 1$. **3**

(f) Show that $\left(2^{4n}-1\right)$ is divisible by 15. **2**

(g) Prove that : **2**

$$\exp\left(\pm i\frac{\pi}{2}\right)=\pm i.$$

Section I

2. (a) The linear disphantine equation $ax + by = c$ has a solution iff d/c, where $d = gcd\ (a, b)$. If x_0, y_0 is any particular solution of this equation, then all other solutions are given by : **8**

$$x = x_0+\left(\frac{b}{d}\right)t,\ y = y_0-\left(\frac{a}{d}\right)t,$$

where t is any arbitrary integer. **8**

(b) Find all solutions in integers of the following equation : **8**

$$56x + 72y = 40$$

3. (a) Use Fermat's little theorem to verify that 17 divides $11^{104} + 1$. **8**

(b) If p is prime, then $(p-1)! \equiv -1 \pmod{p}$. **8**

Section II

4. (a) Define the Mobius μ function on positive integers and show that this function μ is a multiplicative function. **8**

(b) Define Euler's Phi function and prove that :

$$\phi\left(p^k\right) = p^k - p^{k-1}$$

where p is prime & $k > 0$. **8**

5. (a) Find the highest power of 5 dividing $\underline{|1000}$. **8**

(b) State and prove Euler's generalization of Fermet's theorem on integers. **8**

Section III

6. (a) If 'n' has a primitive root, then number of primitive roots of 'n' will be $\phi(\phi(n))$. **8**

(b) If p is an odd prime number and $k \geq 1$, then there exist a primitive root for p^k. **8**

7. (a) Let p be an odd prime and $gcd\ (a,\ p) = 1$, then 'a' is a quadratic residue of p iff : **8**

$$a^{(p-1)/2} \equiv 1 \pmod{p}$$

(b) If p and q are distinct odd primes, then prove that : **8**

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2} \cdot \frac{(q-1)}{2}}$$

where $\left(\frac{p}{q}\right)$ is Legendre symbol notation w.r.t. p and q.

Section IV

8. (a) If $z = x + iy$; where x and y are real, find real and imaginary parts of $\frac{\cos z}{z+1}$. 8

(b) Solve the equation : 8

$$\tan^{-1}\left(e^{ix}\right) - \tan^{-1}\left(e^{-ix}\right) = \tan^{-1}(i),$$

9. (a) If $\tan(A + iB) = x + iy$, prove that : 8

$$x^2 + y^2 + 2x\cot 2A = 1$$

(b) Find the sum of infinity of the series : 8

$$1 - \frac{1}{2}\cos\theta + \frac{1.3}{2.4}\cos 2\theta - \frac{1.3.5}{2.4.6}\cos 3\theta + \ldots\ldots,$$

where $-\pi < \theta < \pi$.