

Roll No. 22311222007
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Exam Code : D-24

Subject Code—62458

B. Sc. EXAMINATION

(Main/Re-appear)

(Batch 2022 Onwards)

(Fifth Semester)

MATHEMATICS

CML-508(i) Paper III

Number Theory and Trigonometry

Time : 3 Hours

Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 9 is compulsory.

Unit I

1. (a) Find the general solution in positive integers of $13x - 17y = 5$. $2 + 13k$ 8
 $53 + 17k$

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P.T.O.

(b) State and prove Wilson's theorem. 8

2. (a) Show that $n^{16} - a^{16}$ is divisible by 85 if n and a are co-prime to 85. 8

(b) Solve the congruences : 8

$$x \equiv 1 \pmod{4}$$

$$x \equiv 3 \pmod{5}$$

and $x \equiv 2 \pmod{7}$ simultaneously.

Unit II $(n+2) = 5187$

$$5187 = (3 \cdot 7 \cdot 13 \cdot 19)$$

3. (a) Show that $\phi(n) = \phi(n+1) = \phi(n+2)$ for $n = 5186$. $\phi(5186) = \phi(2 \cdot 2593)$ 8

(b) Show that $2, 4, 6, \dots, 2m$ is a CRS $(\text{mod } m)$ if m is odd. $m+1 = 1 \pmod{m}$ 8

4. (a) Find the highest power of 9 which divides $365!$ 8

(b) If n is a power of 2, then prove that $\sigma(n)$ is odd. 8

Unit III

5. (a) Show that 20 is a quadratic residue of 31 using Euler's Criterion. 8
(b) Find the solution, if any, of the congruence : 8

$$x^2 \equiv 3 \pmod{5}$$

6. (a) Let p be an odd prime and $(a, p) = 1$. Then a is quadratic residue or a quadratic non-residue $(\text{mod } p)$ according as : 8

$$a^{\frac{p-1}{2}} \equiv 1 \pmod{p}$$

- (b) Find all quadratic residues modulo 11. Also, find the quadratic non-residues of 11. 8

Unit IV

7. (a) State and prove Euler's theorem. 8
(b) If $\tan(A + iB) = x + iy$, prove that : 8
(i) $x^2 + y^2 + 2x \cot 2A = 1$
(ii) $x^2 + y^2 - 2y \coth 2B = -1$

8. (a) If $i^{\alpha+i\beta} = a + ib$, prove that : 8

$$a^2 + b^2 = e^{-(4n+1)\pi\beta}$$

- (b) If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, show that : 8

$$x^2 + y^2 + z^2 + 2xyz = 1$$

Compulsory Question

9. (a) Define Euler's ϕ -function with example. 2

- (b) Find the general value of $\log(-5)$. 2

- (c) Prove that : 2

$$i^i = e^{-(4n+1)\pi/2}$$

- (d) State Chinese remainder's theorem. 2

- (e) Write any two properties of Euler's phi-function. 2

- (f) Show that : 2

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}$$

- (g) Find $d(n)$ and $\sigma(n)$ for $n = 270$. 2

- (h) Define Mobius Inversion formula. 2