Roll No. .... Exam Code : J-22

Subject Code—55642

B. Sc. EXAMINATION

(Main/Re-appear) (Batch 2018 Onwards)

(Fifth Semester)

MATHEMATICS

CML-508(i) (Paper III)

Number Theory and Trigonometry

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question

1. (a) If p is a prime number and p/ab, then either p/a or p/b. 2

(b) Verify Fermat's theorem for $a = 8, p = 3$. 2

(c) If both $\{a_1, a_2, \ldots\ldots, a_{\phi(m)}\}$ and $\{b_1, b_2, \ldots\ldots, b_{\phi(m)}\}$ are RRS (mod m), then : 2

$$\prod_{i=1}^{\phi(m)} a_i = \prod_{i=1}^{\phi(m)} b_i \pmod{m}$$

(d) Evaluate $d(270)$. 2

(e) Is the congruence $x^2 \equiv 150 \pmod{1009}$ solvable ? 2

(f) Prove that : 3

$$\tanh^{-1} x = \sinh^{-1} \frac{x}{\sqrt{1-x^2}}$$

(g) Prove that if 'θ' lies in the interval $-\frac{\pi}{4}$ to $\frac{\pi}{4}$, then : 3

$$\theta = \tan\theta - \frac{1}{3}\tan^3\theta + \frac{1}{5}\tan^5\theta - \ldots\ldots\infty$$

Unit I

2. (a) If a and b are relatively prime and $x = x_0$, $y = y_0$ is a solution of the

equation $ax + by = c$, then all solutions are given by $x = x_0 + kb, y = y_0 - ka$, for integral value of k. 8

(b) State and prove Chinese remainder theorem. 8

3. (a) If p is an odd prime, then show that : 8

$$2^2.4^2.6^2.......(p-1)^2 \equiv (-1)^{\frac{p+1}{2}} \pmod p$$

(b) For an odd positive m, prove that the sum of the integers of any CRS (mod m) is congruent to zero modulo m. 8

Unit II

4. (a) Show that : 8

$$\phi(n) = \phi(n+1) = \phi(n+2) \text{ for } n = 5186$$

(b) Let a be any integer and m be a positive integer. If $(a, m) = 1$, then : 8

$$a^{\phi(m)} \equiv 1 \pmod m$$

5. (a) For any positive real x and y : 8

$$[x - y] \le [x] - [y] \le [x - y] + 1$$

(b) Verify Mobius inversion formula for $n = 24$. 8

Unit III

6. (a) Find the least positive integer (mod 11) to which 335 is congruent. 8

(b) If p is an odd prime, then : 8

$$\left(-\frac{1}{p}\right) = (-1)^{\frac{p-1}{2}} = \begin{cases} 1 & \text{if } p = 1 \pmod 4 \text{ (or } p \text{ is of the form } 4k+1) \\ -1 & \text{if } p = 3 \pmod 4 \text{ (or } p \text{ is of the form } 4k+3) \end{cases}$$

7. (a) If p and q are distinct odd primes, then : 8

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2}\frac{q-1}{2}}$$

(b) State and prove Euler's criterion theorem. 8

Unit IV

8. (a) If $\tan y = \tan\alpha \tanh\beta$ and $\tan z = \cot\alpha \tanh\beta$; prove that : 8

$$\tan(y+z) = \sinh 2\beta \operatorname{cosec} 2\alpha$$

(b) Show that real part of the principal value of $i^{\log(1+i)}$ is $e^{-\frac{\pi^2}{8}} . \cos\left(\frac{\pi}{4}\log 2\right)$. 8

9. (a) With the help of Greogory's series, prove that :

$$\log(1+ix) = \frac{1}{2}\log\left(1+x^2\right) + i\tan^{-1} x$$

hence deduce the expression of $\tan^{-1} x$ in ascending powers of x. 8

(b) If S_n be the sum of n terms of the series $\sin\theta + \sin 2\theta + \sin 3\theta + \ldots\ldots$ to n terms, prove that : 8

$$\lim_{n\to\infty} \frac{S_1 + S_2 + S_3 + \ldots\ldots + S_n}{n} = \frac{1}{2}\cot\frac{\theta}{2}$$