

<https://t.me/BscNonmedical2023>

Roll No. 

Exam Code : D-23

Subject Code—60489

B.Sc. EXAMINATION

(Main/Re-appear) (Batch 2019 Onwards)

(Fifth Semester)

MATHEMATICS

CML-508(i), Paper III

Number Theory and Trigonometry

Time : 3 Hours

Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **9** is compulsory.

Section I

1. (a) Find the remainder when the sum $1+2+3+\dots+1000$ is divided by 8. 8

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(b) Find the least positive incongruent solution of $7x \equiv 5 \pmod{256}$. 8

2. (a) State and prove Fermat's Little Theorem. 8

(b) Find all solutions in positive integers of $5x + 3y = 52$. 8

Section II

3. (a) To show that Euler's function (ϕ) is multiplicative function. 8

(b) Show that $1^2, 2^2, 3^2, \dots, m^2$ is not a CRS $\pmod{m}$ is $m > 2$. 8

4. (a) Find the highest power of 6 contained in $\frac{1500}{8}$.

(b) Prove that : 8

$$d(m, n) = d(m).d(n)$$

$$\text{and } \sigma(m, n) = \sigma(m).\sigma(n) \text{ for } (m, n) = 1$$

Section III

5. (a) State and prove Euler's criterion. 10

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(b) Evaluate : 6

$$\left(\frac{-72}{131} \right).$$

6. (a) State and prove Gauss Reciprocity Law. 10

(b) For all prime to for which $x^2 \equiv 13 \pmod{p}$ is solvable. 6

Section IV

7. (a) If $z = x + iy$, then find the real and imaginary parts of $\frac{\cos z}{(z+1)}$. 8

(b) If $|\cos(\alpha - i\beta)| = 1$; show that : 8

$$\sin^2 \alpha = \sinh^2 \beta.$$

8. (a) To find real part of the principal value of $i^{\log(1+i)}$. 8

(b) To find sum of n terms of series $\cos \alpha + \cos 2\alpha + \cos 3\alpha + \dots$ and deduce the sum of $1^2 + 2^2 + 3^2 + \dots + n^2$. 8

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(Compulsory Question)

9. (a) Write both forms of Gregory Series.

(b) Show that :

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{\pi}{4}.$$

(c) Define order of an integer modulo n .

(d) Explain composite numbers having primitive in brief.

(e) Define Mobius Inversion formula.

(f) Write any *two* properties of Euler's phi-function.

(g) State Chinese remainder's theorem.

(h) Explain statement of Prime Number Theorem.

$$2 \times 8 = 16$$