Roll No. [redacted] Exam Code : D-23

Subject Code—60487

B. Sc. EXAMINATION

(Main/Reappear) (Batch 2019 Onwards)

(Fifth Semester)

MATHEMATICS

CML-507 (i)

Paper–II

Sequences and Series

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) What is interior of $\{3, 4, 5, 6\}$? **2**

(b) Give an example of an infinite set having one limit point. 2

(c) State Cauchy's first theorem on limits. 2

(d) Discuss the convergence of the series

$$\sum\left(\frac{1}{n}\right)^{1/n}.$$ 2½

(e) State Leibnitz's test for the convergence of the alternating series. 2½

(f) Find the Fourier expansion of $f(x) = x$ in $[-\pi, \pi]$. 2½

(g) If f is a bounded function of $[a, b]$ and P_1 is a refinement of a partition P of $[a, b]$, then show that :

$$U(f, P_1) \leq U(f, P).$$ 2½

Unit I

2. (a) Prove that the intersection of a finite number of open set is an open set. 8

(b) Prove that A set is closed iff it contains all its limit points. 8

3. (a) Show that :

$$\lim_{n\to\infty}\left[\frac{1}{n^2}+\frac{1}{(n+1)^2}+.....+\frac{1}{(2n)^2}\right]=0.$$ 8

(b) Prove that a sequence converges if and only if it is a Cauchy sequence. 8

Unit II

4. (a) Test the convergence of the series :

$$1+\frac{x^2}{2}+\frac{x^4}{4}+\frac{x^6}{6}+......,\ x>0.$$ 8

(b) Test the convergence of the series :

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}+\frac{4!}{5^4}x^4+....(x>0).$$ 8

5. (a) Discuss the convergence and absolute convergence of the series :

$$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}\left(\sqrt{n+1}-\sqrt{n-1}\right).$$ 8

(b) Test the convergence of the series :

$$\sum_{n=3}^{\infty} \frac{\left(n^3+1\right)^{1/3}-n}{\log n}. \qquad 8$$

Unit III

6. (a) Find the Fourier series for the function :

$$f(x)=|\sin x|, -\pi < x < \pi. \qquad 8$$

(b) Obtain the Fourier expansion of the function defined by :

$$f(x)=\begin{cases} 1+\dfrac{2\pi}{x}, & \text{when } -\pi \le x \le 0 \\ 1-\dfrac{2\pi}{x}, & \text{when } 0 \le x \le \pi \end{cases}.$$

Hence deduce that :

$$\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots=\frac{\pi}{8}. \qquad 8$$

7. (a) Find the half-range cosine series for $f(x)=x(\pi-x)$ in the interval $(0, \pi)$. Hence deduce that :

$$\frac{\pi^2}{6}=\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots \qquad 8$$

(b) Given the series $x=2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin nx$, show that $\sum_{n=1}^{\infty}\frac{1}{n^2}=\frac{\pi^2}{6}$ using Parseval's identity. 8

Unit IV

8. (a) Prove that a continuous function 'f' on $[a, b]$ is integrable on $[a, b]$. Is the converse true ? Justify. 8

(b) By definition, prove that :

$$\int_0^a \cos x\, dx=\sin a.$$

where a is a fixed number. 8

9. (a) If 'f' is integrable on $[a, b]$ and F is the primitive of f on $[a, b]$, then :

$$\int_a^b f\,dx = F(b) - F(a).$$ 8

(b) Prove that inequality :

$$\frac{\sqrt{3}}{8} \leq \int_{\pi/4}^{\pi/3} \frac{\sin x}{x}\,dx \leq \frac{\sqrt{2}}{6}.$$ 8