Roll No. 22311232008807 Exam Code : D-24

Hari

Subject Code—62456

B. Sc. EXAMINATION

(Main/Reappear) (Batch 2022 Onwards)

(Fifth Semester)

MATHEMATICS

Paper–II

CML-507 (i)

Sequence and Series

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Section. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Explain interior point of a set. 2

(b) Every monotonically increasing sequence unbounded above diverges to ∞. 2

(c) Show that the series. 2

(d) State logarithmic test $\sum_{n=1}^{\infty}\frac{n}{2n+1}$ is not convergent. 2

(e) Define periodic functions. 2

(f) Explain Mean value theorem. 2

(g) Defined oscillatory sum. 2

(h) Define lower sum. 2

Section I

2. (a) Prove that a set is closed if and only if it contains all its limit points. 8

(b) State and prove Bolzano-Weierstrass theorem. 8

3. (a) By definition, show that :

$$\lim_{n\to\infty} \frac{n^2+1}{2n^2+3} = \frac{1}{2}.$$ 8

(b) Prove that the sequence $\langle a_n \rangle$ defined by :

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots\dots + \frac{1}{n}$$

is not convergent. 8

Section II

4. (a) State and prove Cauchy's General Principle of Convergence. 8

(b) Discuss the convergence of the series

$$\sum_{n=2}^{\infty} \left[\log(n+1) - \log(n)\right].$$ 8

5. (a) Test the convergence of series :

$$1 + \frac{3}{7}x + \frac{3\cdot 6}{7\cdot 10}x^2 + \frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3 + \frac{3\cdot 6\cdot 9\cdot 12}{7\cdot 10\cdot 13\cdot 16}x^4 \dots\dots$$

8

(b) Test for convergence of the series :

$$1^p + \left(\frac{1}{2}\right)^p + \left(\frac{1\cdot 3}{2\cdot 4}\right)^p + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^p + \ldots\ldots. \quad 8.$$

Section III

6. (a) Expand $f(x) = x\sin x$ as Fourier series in $(0, 2\pi)$. 8

(b) Obtain the Fourier expansion of the function $f(x)$, where :

$$f(x) = \begin{cases} -\pi & \text{when } -\pi < x < 0 \\ x, & \text{when } 0 < x < \pi \end{cases}.$$

Hence deduce that :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots \frac{\pi^2}{8}. \quad 8$$

7. (a) Find the half-range cosine series for the function $f(x) = x(\pi - x)$ in $(0, \pi)$. Also deduce that :

$$\frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{5^3} + \ldots\ldots \frac{\pi^3}{32}. \quad 8$$

(b) Let :

$$f(x)=\begin{cases}-1 & \text{when } -\pi < x < 0 \\ 1, & \text{when } 0 < x < \pi\end{cases}.$$

Using Parseval's identity, compute the sum $\sum_{k=1}^{\infty}(2k-1)^{-2}$. 8

Section IV

8. (a) A bounded function f is integrable on $[a, b]$ if and only if for each $\epsilon > 0$, there corresponds a $\delta > 0$ such that for every partition P of $[a, b]$ with $\|P\| < \delta$, then $U(f, P) - L(f, P) < \epsilon$. 8

Let m, M, M', m' are g.l.b & l.u.b respectively

$m \le f(x) \le g(x) \le M'$

$m \le m'$ $M \le M'$

Consider $P = \{a, x_0, x_1, \ldots x_n\}$ for $[a, b]$. f & g are bounded on $[a, b]$

$\therefore [x_{r-1}, x_r)$

$r = (1, 2, 3 \ldots n)$

(b) Show that :

$$\int_1^2 x\,dx = \frac{3}{2}.$$

9. (a) If f and g are bounded and integrable functions on $[a, b]$ with $f(x) \le g(x)$, $x \in [a, b]$, then $\int_a^b f\,dx \le \int_a^b g\,dx$. 8

(b) Prove the inequality

$$\frac{1}{2} \le \int_0^1 \frac{dx}{\sqrt{4 - x^2 + x^3}} \le \frac{\pi}{6}.$$ 8

Put $n \to \infty$ It will form $\frac{\infty}{\infty}$ form

Apply L-Hospital Rule Diff^n Upper & Lower

Then $\cancel{\frac{\infty^2+1}{2\infty^2+3}} = \frac{\infty}{\infty}$ $\frac{2n}{2(2)n}$

Put $n = \infty$ Again form $\frac{\infty}{\infty}$ form which is Indermined form

So again apply L-Hospital Rule

Then $\lim_{n \to \infty} \frac{2}{4} = \frac{1}{2}$

Hence Proved.