Roll No. ..2231123200007   Exam Code : M-25

Subject Code—64497

B.Sc. EXAMINATION

(Batch 2022 Onwards)

(Sixth Semester)

MATHEMATICS

CML-605 (i)

Linear Algebra

Time : 3 Hours   Maximum Marks : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Prove that L(S) = S iff S is a subspace of V. 2

(b) In any vector space V(F), prove that $au = bu$ and $u \neq 0$ implies that $a = b$, where $a, b \in F$ and $u \in V$. 2

(c) Define kernel of a linear transformation. 2

(d) Let $T : U \to V$ be a linear transformation. Prove that :

$T(u - v) = T(u) - T(v) \ \forall u, v \in U$. 2

(e) Show that the linear transformation $T : R^2 \to R^3$ defined by $T(x, y) = (x + y, x - y, y)$ is a non-singular linear transformation. 2

(f) Define an eigen value and an eigen vector of a linear transformation. 2

(g) Let V be an inner product space. Show that if $b, c \in F$ and $u, v, w \in V$ are arbitrary, then $< u, bv + cw > = \bar{b} < u, v > + \bar{c} < u, w >$. 2

(h) Prove that $\frac{v}{\|v\|}$ is always a unit vector. 2

Unit I

2\. (a) A non-empty subset W of a vector space V(F) is a subspace of V iff $au + bv \in W$ for $a, b, \in F$ and $u, v \in W$. 8

(b) For what value of k will the vector $u = (1, k, 5)$ in $V_3(R)$ be a linear combination of vectors $v = (1, -3, 2)$ and $w = (2, -1, 1)$. 8

3\. (a) If W is a subspace of $V_3(R)$ generated by $\{1, 0, 0), (1, 1, 0)\}$, find V/W and its basis. 8

(b) If S is linearly independent and $v \notin <S>$, then the set $S \cup \{v\}$ is linearly independent. 8

Unit II

4\. (a) Show that the function $T : R^2 \to R^3$ given by $T(x, y) = (x + 2y, 3x - 5y, y)$ is a linear transformation. 8

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(b) Find a linear transformation $T: R^3 \to R^2$ such that $T(1, 1, 1) = (1, 0)$ and $T(1, 1, 2) = (1, -1)$. Also, verify your answer. 8

5. (a) If $T : U(F) \to V(F)$ is a linear transformation, then $\dim(R(T)) + \dim(N(T)) = \dim U$. 8

(b) If $T: R^4 \to R^3$ is a linear transformation defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3) = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$, then verify that $\rho(T) + \mu(T) = \dim R^4 = 4$. 8

Unit III

6. (a) Let $T: U \to V$ be invertible linear transformation and $T^{-1}: V \to U$ be its inverse. Prove that T^{-1} is a linear transformation. 8

(b) Find the matrix representing the transformation $T : R^3 \rightarrow R^4$ defined by $T(x, y, z) = (x + y + z, 2x + z, 2y - z, 6y)$ relative to the standard basis of R^3 and R^4. 8

7. (a) Let $B = \{u_1, u_2, \dots, u_n\}$ be a basis of vector space U(F) and $T : U \rightarrow U$ be a linear transformation. Then for any vector $u \in U$, $[T(u), B] = [T, B][u, B]$. 8

(b) Let $T : R^3(R) \rightarrow R^3(R)$ be a linear transformation defined by $T(x, y, z) = (2x - y, x + y + z, 2z)$. Find the characteristic and minimal polynomial for T. 8

Unit IV

8. (a) Let V be an inner product space. Then $|\langle u, v \rangle| \leq \|u\| \, \|v\|$ for all $u, v \in V$. 8

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(b) Let V(R) be a real inner product space. Let $\alpha, \beta \in V$ such that $\alpha \perp \beta$. Then show that $\|\alpha+\beta\|^2 \|\alpha\|^2 + \|\beta\|^2$. Also, prove the converse. **8**

9. (a) If $\{u_1, u_2, \ldots . u_n\}$ be an othonormal subset of an inner product space V(F), then prove that $\sum_{i=1}^{n} |\langle u, u_i \rangle|^2 \leq \|u\|^2 \ \forall u \in V$. **8**

(b) Obtain an orthonormal basis of $V_3(R)$ with standard inner product defined on it, given the basis $u_1 = (1, 1, 1)$, $u_2 = (1, -2, 1)$, $u_3 = (1, 2, 3)$. **8**

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