Roll No. [redacted] Exam Code : M-24

Subject Code—61378

B.Sc. EXAMINATION

(Main/Reappear) (Batch 2019 Onwards)

(Sixth Semester)

MATHEMATICS

CML-605 (i)

Linear Algebra

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. **1.** is compulsory. All questions carry equal marks.

1. (a) Show that any field forms a vector space over itself. 3

(b) Find the condition on u, v, w so that (u, v, w) belongs to the vector space generated by $(1, 2, 3)$ and $(-1, 2, 4)$. 3

(c) Determine whether the following mapping is a linear transformation or not ? 2
$T : R^2 \to R^3$ defined by $T(x, y) = (x + 1, 2y, x + y)$.

(d) Show that a linear transformation $T : U \to V$ is a non-singular if and only if T is one-one. 2

(e) Prove that the minimal polynomial of a linear operator T divides every polynomial which has T as a zero. 2

(f) Let T be a linear operator on an inner product space V(F). Then prove that $T = 0$ if and only if $< T(\alpha), b > = 0$ for all $a, b \in V$. 2

(g) Define orthonormal set and also give an example of an orthonormal set. 2

Unit I

2. (a) Prove that union of two subspaces is a subspace if and only if one is contained in the other. 8

(b) Determine a basis of the sub-space spanned by the vectors $(-3, 1, 2)$, $(0, 1, 3)$, $(2, 1, 0)$ and $(1, 1, 1)$. 8

3. (a) Define quotient space and also prove that if W is a subspace of a finite dimensional vector space V(F), then

$$\dim\left(\frac{V}{W}\right) = \dim V - \dim W,$$ where dim V denotes the dimension of V. 8

(b) Show that $\{u_1, v_1, w_1\}$ is a basis of $R^3(R)$ and also find the co-ordinates u with respect to u_1, v_1 and w_1, where $u = (4, 5, 6)$; $u_1 = (-1, 1, 1)$, $v_1 = (-1, 1, 0)$ and $w_1 = (1, 0, -1)$. 8

Unit II

4. (a) Show that the linear transformation $T : R^2 \to R^2$ defined by $T(x, y) = (x \cos\theta + y \sin\theta, -x \sin\theta + y \cos\theta)$ is a vector space isomorphism. 8

(b) Let $T : U \to V$ be an onto linear transformation. Then prove that : 8

$$\frac{U}{\operatorname{Ker} T} \cong V.$$

5. (a) Find a linear transformation $T : R^3 \to R^4$ whose range is generated by $(1, 2, 0, -4)$ and $(2, 0, -1, -3)$. 8

(b) Check whether the linear transformation $T : R^3 \to R^2$ is one to one, onto or both : 8

$$T(x, y, z) = (x + y, y + z).$$

Unit III

6. (a) Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{bmatrix}$. 8

(b) A linear transformation, $T : R^3 \to R^3$ is defined by $T(x_1, x_2, x_3) = (x_1 + 2x_2, 3x_3, x_2 - 2x_3)$. Find the matrix associated with the basis $\{(1, 1, 1), (0, 1, 1), (0, 0, 1)\}$. 8

7. (a) Prove that the minimal polynomial of a linear operator is unique. 8

(b) Let $X = (1, 2, 1)$ be relative to the standard basis. Find its coordinates relative to basis $\{(1, 1, 0), (1, 0, 1), (1, 1, 1)\}$ using the change of basis matrix. 8

Unit IV

8. (a) Let V be an inner product space. Then prove that $|\langle u, v \rangle| \leq \|u\| \, \|v\|$ for all $u, v \in V$. 8

(b) Using Gram-Schmidt orthogonalization process, construct an orthonormal basis of $V_3(R)$ with standard inner product defined on it, given the basis :

$u_1 = (1, 1, 1), u_2 = (1, -2, 1), u_3 = (1, 2, 3)$. 8

9. (a) Let T be a linear operator on a finite dimensional inner product space V(F). Show that T can be uniquely expressed as a sum of self adjoint and skew-symmetric operator. 8

(b) If V is an inner product space, then prove that $\| u + v \| \leq \| u \| + \| v \|$ for all $u, v \in V$. 8