$(1, -1, 2)$ to be basis of R^3, let $T : R^3 \to R^2$ be the linear transformation such that $T(v_1) = (1, 0)$ $T(v_2) = (0, 1)$, $T(v_3) = (1, 1)$. Find T. **8**

(b) Prove that finite dimensional vector space over the same field are isomorphic if and only if they have the same dimension. **8**

Section III

6. (a) Let $T_1 : U \to V$ and $T_2 : V \to W$ be two linear transformation. Then prove that :

(i) if T_2T_1 is one-one, then T_1 is one-one

(ii) if T_2T_1 is onto then T_2 is onto. **8**

(b) Let T be the linear operator on R^2 defined by $T(x, y) = (4x - 2y, 2x + y)$. Find the matrix of T with respect to the basis $B = \{(1, 1), (-1, 0)\}$. Also verify that $[T, B]\,[u, B] = [T(u), B]$. **8**

Subject Code—53880

B. Sc. EXAMINATION

(Batch 2018 Onwards)

(Main)

(Sixth Semester)

MATHEMATICS

CML-605(*i*)

Linear Algebra

Time : 3 Hours *Maximum Marks* : 80

Note : Attempt *Five* questions in all. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Define Inner product space. **3**

(b) Prove that $\frac{v}{\|v\|}$ is always a unit vector. **3**

(c) Define orthogonal complement of a subspace W of V. **3**

(d) Define basis of a vector space. **3**

(e) Prove that intersection of two subspaces of a vector space is a subspace. **4**

Section I

2. (a) Prove that necessary and sufficient condition for a non-empty subset W of vector space V(F) to be subspace of V is that W is closed under addition and scalar multiplication. **8**

(b) Prove that necessary and sufficient condition for a vector space V(F) to be a direct sum of its subpsaces W_1 and W_2 are that : **8**

(i) $V = W_1 + W_2$

(ii) $W_1 \cap W_2 = \{0\}$

3. (a) Show that the set $\{(1, i, 0), (2i, 1, 1), (0, 1 + i, 1 - i)\}$ is a basis for $V_3(C)$ and determine the co-ordinates of the vectors (1, 0, 0), (0, 1, 0), (0, 0, 1) with respect to the basis. **8**

(b) If W_1 and W_2 are subspaces of V(F), $\dim W_1 = 4$, $\dim W_2 = 5$ and $\dim V = 7$ then find the possible value of dim $(W_1 \cap W_2)$. **8**

Section II

4. (a) For a linear transformation $T : R^2 \to R^3$ such that :

$$T(x_1, x_2) = (x_1 - x_2, x_2 - x_1, -x_1),$$

find a basis and dimension of its range space and its null space. Also verify that rank (T) + nullity (T) = dim R^2. **10**

(b) If $T : V \to V$ is a linear transformation, then prove that following statements are equivalent : **6**

(i) $R(T) \cap N(T) = \{0\}$

(ii) if $T(T(v)) = 0$ then $T(v) = 0$, $v \in V$.

9. (a) Let T be linear operator on an inner product space V(F). Then prove that following statements are equivalent : **10**

(i) $T^* = T^{-1}$ where T^* is adjoint of T

(ii) $TT^* = T^*T = I$

(iii) $<T(u), T(u)> = <u, v>$ for all $u, v \in V$

(iv) $\|T(u)\| = \|u\|$ for all $u \in V$.

(b) Let T be a normal operator on an inner product space V. If $u \in V$ then show that $T(u) =$ iff $T^*(u) = 0$. **6**

7. (a) Let T be a linear operation of R^4 and matrix representation of T with respect to some ordered basis be :

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}.$$

Find the minimal polynomial of T. **8**

(b) Let T be a linear operator on a finite dimensional vector space V(F). Let $\lambda_1, \lambda_2, ...\lambda_n$ be distinct eigen values of T and $v_1, v_2, ... v_n$ be corresponding eigen vectors of T. Then prove that $\{v_1, v_2, ... v\}$ is linearly independent. **8**

Section IV

8. (a) State and prove Bessel's Inequality. **10**

(b) Let V(F) be an inner product space. If $u, v \in V$ such that $|<u, v>| = \|u\| . \|v\|$, then show that u and v are linearly dependent. **6**