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Roll [redacted] Exam Code : M-24

Subject Code—61379

B. Sc. EXAMINATION

(Main/Reappear) (Batch 2019 Onwards)

(Sixth Semester)

MATHEMATICS

CML-606 (i)

Mechanics–II

Time : 3 *Hours* *Maximum Marks* : 80

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. Q. No. 1 is compulsory. All questions carry equal marks.

1. (a) Define friction, force of friction and coefficient of friction.

   (b) Find the centre of gravity of a thin uniform rod.

(c) If t_1 and t_2 are the periods corresponding to two different attached weights and c_1, c_2 the statical extensions due to these weights, then prove that :

$$g = \frac{4\pi^2(c_1 - c_2)}{t_1^2 - t_2^2}.$$

(d) Define work. Also write the expression for work done in stretching an elastic string.

(e) Write the expressions for velocity and reaction at any point P when a particle is projected with velocity u from the lowest point and moves along the inside of a smooth vertical circle. Also discuss the case when velocity vanishes before reaction has vanished.

(f) Write the expressions for velocity and reaction at any point P when a particle slides down the arc of a smooth cycloid, whose axis is vertical and vertex downwards.

(g) A particle is projected with a velocity of 14 m/sec at an angle of elevation 60°. Find the time of flight and greatest height of the projectile.

(h) A particle is projected with a velocity of 80 ft/sec at an angle of 45° to the horizontal. Find its range on a plane inclined at 30° to the horizontal when projected up the plane.

Unit I

2. (a) A square of side $2a$ is placed with its plane vertical between two smooth pegs which are in the same horizontal line and at a distance c apart. Show that it will be in equilibrium when the inclination of one of its sides to the horizontal is either 45° or $\frac{1}{2}\sin^{-1}\left(\frac{a^2 - c^2}{c^2}\right)$.

(b) A uniform rod rests in a vertical plane within a fixed hemispherical bowl whose radius is equal to the length of the rod. If μ be the coefficient of friction between the rod and the bowl, then show that in limiting equilibrium the inclination of the rod to the horizontal is $\tan^{-1}\left(\frac{4\mu}{3-\mu^2}\right)$.

3. (a) A uniform rod AB of weight W rests with its ends A and B on two smooth planes inclined to the horizontal at angles of 45° and 30° respectively, in such a way that the vertical plane through the rod is perpendicular to both the inclined planes. Find what weight fixed to the rod at a quarter of its length from the end B will suffice to enable the rod to rest horizontally.

(b) A uniform circular lamina of radius $3a$ and centre O has a hole in the form of an equilateral triangle of side $2a$ with one vertex at O. Prove that the distance of the centre of gravity from O is $\frac{2a}{9\pi-\sqrt{3}}$.

Unit II

4. (a) An elastic string whose modulus of elasticity is λ, is stretched to double its length and is tied to two fixed points distant $2a$ apart. A particle of mass m, tied to its middle point is displaced along the line of the string through a distance equal to half its distance from the fixed point and released. Prove that the time of complete oscillation is $\sqrt{\frac{am}{\lambda}}$ and maximum velocity acquired is $\sqrt{\frac{a\lambda}{m}}$.

(b) A mass of 4 kg suspended from a light elastic string of natural length 3 m extends it a distance 2 m. One end of the string is fixed and a mass of 2 kg is attached to the other. The mass is held so that the string is just unstretched and is then let go. Find the amplitude, the period and the maximum velocity of the motion.

5. (a) State and prove principle of work and energy.

(b) A train of mass 200 tons is ascending an incline of 1 in 100, the resistance to the motion being 15 lbs wt. per ton. When its speed has reached 12 m.p.h., then find its acceleration if the horse power developed is 600.

Unit III

6. (a) A particle is projected from the lowest point inside a smooth sphere of radius a with velocity $2\sqrt{ag}$. Find the point at which it will leave the sphere and equation to subsequent path of the particle.

(b) A particle slides down a rough curve under gravity in a vertical plane. Discuss the motion.

7. (a) A particle slides down the outside of a smooth vertical circle starting from rest at the highest point. Discuss the motion.

(b) A particle is placed very close to the vertex of a smooth cycloid whose axis is vertical and vertex upwards and is allowed to run down the curve. Discuss its motion.

Unit IV

8. (a) Find the velocity and direction of projection of shot which passes in a horizontal direction over the top of a wall 64 ft. high and 192 ft. distant from the gun.

(b) A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection, then prove that $\tan\theta = \tan\alpha + \tan\beta$.

9. (a) If at any point of a parabolic path velocity be u and the inclination to the horizon be α, then show that the particle is moving at right angles to this direction after time $\frac{u}{g}\operatorname{cosec}\alpha$.

(b) If R is the maximum range on an inclined plane through the point of projection of a particle and T the corresponding time of flight, then show that $R = \frac{1}{2}gT^2$.